Finding New Physics using heavy flavor decays
What is Heavy Flavor Physics?

- Define Heavy Flavor Physics
  - Flavor Physics: Study of interactions that differ among flavors: (quark flavors are u, d, c, s, b, t)
  - Heavy: Not SM neutrino’s or u or d quarks, maybe s quarks, concentrate here on b quarks (some c), t too heavy

- u, d, ν’s
  - too light
- S, μ
  - maybe
- c & b, τ; ν_M’s?
  - just right
- t
  - too heavy
Physics Beyond the Standard Model

- **Baryogenesis:** From current measurements can only generate \((n_B - \bar{n}_B)/n_\gamma \approx 10^{-20}\) but \(~6 \times 10^{-10}\) is needed. Thus New Physics must exist to generate needed CP Violation

- **Dark Matter**

- **Hierarchy Problem:** We don’t understand how we get from the Planck scale of Energy \(~10^{19}\) GeV to the Electroweak Scale \(~100\) GeV without “fine tuning” quantum corrections

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12 orders of magnitude differences not explained; t quark as heavy as Tungsten
Formalism

- Standard model fermions
  \[
  \left( \begin{array}{ccc}
  u_L & c_L & t_L \\
  d_L & s_L & b_L \\
  e^-_L & \mu^-_L & \tau^-_L \\
  \nu_e & \nu_{\mu} & \nu_{\tau}
  \end{array} \right), \quad u_R, \ d_R, \ c_R, \ s_R, \ t_R, \ b_R
  \]

- SM gauge bosons: \(\gamma, W^\pm, Z^0, H^0\).

- Lagrangian for charged current interactions is
  \[
  L_{cc} = -\frac{g}{\sqrt{2}} J^\mu_{cc} W^\dagger_\mu + \text{h.c.},
  \]

- where
  \[
  J^\mu_{cc} = (\bar{\nu}_e, \bar{\nu}_{\mu}, \bar{\nu}_{\tau}) \gamma^\mu V_{MNS} \left( \begin{array}{c} e_L \\ \mu_L \\ \tau_L \end{array} \right) + (\bar{u}_L, \bar{c}_L, \bar{t}_L) \gamma^\mu V_{CKM} \left( \begin{array}{c} d_L \\ s_L \\ b_L \end{array} \right)
  \]
Consider the charm quark. It forms a 2nd generation doublet with the strange quark (c,s). Yet it also decays into the d quark which is in the first generation with the u quark (u,d).

We say this happens because the s & d quarks are “mixed” i.e. their wave functions really are described by a rotation matrix

\[
\begin{pmatrix}
  d' \\
  s'
\end{pmatrix} =
\begin{pmatrix}
  \cos \theta_c & \sin \theta_c \\
  -\sin \theta_c & \cos \theta_c
\end{pmatrix}
\begin{pmatrix}
  d \\
  s
\end{pmatrix} =
\begin{pmatrix}
  V_{ud} & V_{us} \\
  V_{cd} & V_{cs}
\end{pmatrix}
\begin{pmatrix}
  d \\
  s
\end{pmatrix}
\]

where the s' couples to c
Quark Mixing & CKM Matrix

- All 3 generations of -1/3 quarks (d, s, b) are mixed
- Described by CKM matrix (also $\nu$ are mixed)

$$ V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \lambda^2 / 2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2 / 2 - \lambda^4(1 + 4A^2) / 8 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 + A\lambda^4(1/2 + (\rho - i\eta)) & 1 - A^2\lambda^4 / 2 \end{pmatrix} $$

- Unitary 3x3 matrix can be described by 4 parameters $\lambda = 0.225$, $A = 0.8$, constraints on $\rho$ & $\eta$
- These are fundamental constants of nature in the Standard Model

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Why these values? Are the two related? Are they related to masses?
Y, formed of $b\bar{b}$ quarks, found at Fermilab in the $\mu^+\mu^-$ channel

Followed by Doris Y, Y2; CLEO & CUSB that distinctly observed all 3 states, & published on the 1979 Xmas card

A bit of history

Herb et. al, PRL 39, 252 (1977)

GREETINGS FROM CESR

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Discovery of Y(4S)

- The Y states were narrow, their observed widths were consistent with the experimental mass resolution, so below the threshold to decay into B\bar{B}
- Another resonance was found that was \sim 20 \text{ MeV} wide, & subsequently shown to decay into either B^+B^- or B^0\bar{B}^0
B Experiments

- $e^+e^-$ at $Y(4S)$ ARGUS, CLEO, BaBar, & Belle
- $e^+e^-$ at $Z^0$, LEP & SLC
- CDF & D0, 1.8 TeV $p\bar{p}$
- LHCb, CMS & ATLAS, 7-8 TeV $pp$
e⁺e⁻ at Y(4S)

- All detectors have cylindrical geometries with common elements.
- Key: PID, CsI ecal.
- Vertex detector usually Si strips, to measure B & B vertex separations, possible since beams in Belle & Babar have different energies; causes boost along beam direction. Typical resolutions on $\tau_B \sim 900$ fs.
The LHC

- \(4\ \text{TeV} \times 4\ \text{TeV}\) pp collisions (future \(\sim 7 \times \sim 7\))
27 km in circumference
The LHCb Detector
Detector Geometry

- Complementary to ATLAS & CMS
- Much less expensive
The Forward Direction at the LHC

- The primary pp collision produces a pair of $b\bar{b}$ quarks. They then form hadrons. In the forward region at LHC the $b\bar{b}$ production $\sigma$ is large.

- The hadrons containing the $b$ & $\bar{b}$ quarks are both likely to be in the acceptance. Essential for knowing if a neutral B meson started out as a $B^0$ or $\bar{B}^0$, determined by “flavor tagging”.

- At $\mathcal{L}=2\times10^{32}/\text{cm}^2\cdot\text{s}$, we get $\sim10^{12}$ B hadrons in $10^7$ sec.

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LHCb detector ~ fully installed and commissioned → walk through the detector using the example of a $B_s \rightarrow D_s K$ decay
**B-Vertex Measurement**

**Example: \( B_s \rightarrow D_s K \)**

- **47 \( \mu m \)**
- **144 \( \mu m \)**
- **440 \( \mu m \)**
- **K\(^+\)**
- **K\(^+\)**
- **K\(^-\)**
- **\( d \sim 1 \text{cm} \)**
- **Decay time resolution = 40 fs**

**Vertex Locator (Velo)**

Silicon strip detector with

- \( \sim 5 \mu m \) hit resolution
- \( \rightarrow 30 \mu m \) IP resolution

**Vertexing:**
- trigger on impact parameter
- measurement of decay distance & decay time = \( d / v = m d / p \)

**Double Gaussian fit**

\[ \sigma_1 = 33 \pm 1 \text{ fs} \]
\[ \sigma_2 = 67 \pm 3 \text{ fs (31\%)} \]

\[ \sigma(\tau) \sim 40 \text{ fs} \]
Momentum and Mass measurement

Momentum meas. + direction (VELO):
Mass resolution for background suppression

Mass resolution $\sigma \sim 15$ MeV

$b_{\text{tag}}$

Primary vertex

$B_{s} \rightarrow D_{s} K^{+}$

$B_{s} \rightarrow D_{s} K^{+}$, $K^{+}$, $K^{-}$, $\pi^{+}$, $K^{+}$

$B_{s} \rightarrow D_{s} K^{+}$

$B^{0} \rightarrow D^{+} K^{-}$

$B^{0} \rightarrow D^{+} K^{-}$

$B^{0} \rightarrow D^{+} K^{-}$, $K^{+}$, $K^{-}$, $\pi^{+}$, $K^{+}$

$B^{0} \rightarrow D^{+} K^{-}$

$B^{0} \rightarrow D^{+} K^{-}$

$m(D_{s}K) (\text{MeV})$
**Hadron Identification**

**RICH: K/π identification using Cherenkov light emission angle**

- **RICH1**: 5 cm aerogel $n=1.03$
  
- **RICH2**: 100 m$^3$ CF$_4$ $n=1.0005$

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- **B_s → D_s K**
- **SS flavour tagging**
  
  - $\pi^+, K^+$
  - $K^+$
  - $K^-$
  - $\pi^-$

- **Primary vertex**
- **b_{tag}**

- **Kaon identification performance**

  - $K \rightarrow K : 96.77 \pm 0.06\%$
  - $\pi \rightarrow K : 3.94 \pm 0.02\%$
Calorimetry and L0 trigger

Calorimeter system:
- Identify electrons, hadrons, $\pi^0, \gamma$
- Level 0 trigger: high $E_T$ electron and hadron

ECAL (inner modules): $\sigma(E)/E \sim 8.2\% / \sqrt{E} + 0.9\%$
Muon identification and L0 trigger

Muon system:
- Level 0 trigger: High $P_t$ muons
- OS flavour tagging

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Triggering

Trigger is crucial as $\sigma_{b\bar{b}}$ is less than 1% of total inelastic cross section and $B$ decays of interest typically have $B$ branching ratios of $<10^{-5}$.

Hardware level (L0)

Search for high-$p_T$ $\mu$, $e$, $\gamma$ and hadron candidates.

Software level (High Level Trigger, HLT)

Farm with $\Theta(29000)$ multi-core processors.

Very flexible algorithms, writes $\sim5$ kHz to storage.

This is the bottleneck.
Detector Performance

- Detector works better than expected
- Run at $4 \times 10^{-32} \text{ cm}^{-2}/\text{s}$ instead of $2 \times 10^{32}$, with fewer bunches in the machine which is more difficult $\sim <1.5>$ interactions/crossing
- Detector efficiency $>95\%$ for all systems
- Problems: Vertex resolution slightly worse, flavor tagging somewhat poorer
- Luminosity is leveled – small changes of $L$ with time; beams are brought closer together when currents decrease

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Luminosity Leveling

- Luminosity is maintained as at a constant value of \( \sim 4 \times 10^{32} / \text{cm} \cdot \text{s} \) by displacing beams transversely.
- Integral L is 1/fb in 2011, collected 2/fb more in 2012.
B$^-$ → J/ψ K$^-$

LHCb Event Display
20 MHz of bunch crossing (in 2012, with 50 ns bunch spacing) with an average of $1.5 \, \text{pp}$ interactions per bunch crossing $\Rightarrow$ this level of pileup not an issue for LHCb
Consider the b decay of $\mu^- \rightarrow e^- \bar{v}_e v_\mu$

The decay width is given by

$$\Gamma_\mu = \frac{G_F^2}{192\pi^3} m_\mu^5 \times (\text{phase space}) \times (\text{radiative corrections})$$

Since $\Gamma_\mu \Theta_\mu = \hbar$, measuring the muon lifetime determines $G_F$. 

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|V_{us}| = 0.97418\pm 0.00026

is measured using nuclear $\beta$ decays

For $|V_{us}|$ use semileptonic kaon decays. The decay width is given by

$$\Gamma(K_{l3}) = \frac{C_K^2 G_F^2 M_K^5}{192\pi^3} S_{EW} |V_{us}|^2 f_+(0)^2 \times I_{K,l}(\lambda) \left(1 + 2\Delta^{SU(2)}_K + 2\Delta^{EM}_{K,l}\right)$$

- $C_K$ is a Clebsch-Gordan coefficient = $1/2$
- $S_{EW}$ is the short-distance EW correction = 1.0232
- $\Delta$’s are SU(2) breaking & long-distance E&M corrects
- $I_{K,l}(\lambda)$ is the phase space integral
- $f_+(0)$: Here we have quark transition, yet the quarks have to form a single hadron, the $\pi^0$.
- The probability of this happening is parameterized in terms of the 4-momentum transfer squared, $q^2=(p-p')^2$. From the fact that the $K\rightarrow\pi$ weak transition must be Vector

\[
\langle \pi(p') | V_\mu = \gamma_\mu (1+\gamma_5) | K(p) \rangle = (p_\mu + p'_\mu) f_+(q^2) + (p_\mu - p'_\mu) f_-(q^2)
\]
- For massless leptons the $f_-(q^2)$ term vanishes.
- The shape of $f(q^2)$ can be measured, so only $f_+(0)$ remains to be calculated.
Measurements of $f_{+}(0)|V_{us}|$

- $f_{+}(0)=0.964(5)$
- $\lambda=|V_{us}|=0.2246\pm0.0012$

Experiment measures $K$ lifetime, shape of form-factor & value of the form-factor at $q^2=0$
Basic decay diagram:

- Two methods used to determine $|V_{cb}|$ from data: **Exclusive**, only a D or D* produced, & **Inclusive**, take all $b \rightarrow c$ decays
- If $B \rightarrow D$ one form-factor, for $B \rightarrow D^*$, have 3
Based on HQET invented by N. Isgur & M. Wise

- Idea is that there are spin & flavor symmetries between two $\infty$ heavy quarks; the b & c quarks are not quite that heavy, but corrections can be calculated in a controlled way. In HQET only 1 ff for $B \to D^*$, where there are 3 independent spin states.

- Consider the invariant 4-velocity transfer, $\omega$. When $\omega = 1$, the b transforms into a c with the same velocity, so the form-factor is unity modulo some small corrections.

- Note $\omega = \left( m_B^2 + m_{D^{(*)}}^2 - q^2 \right) / \left( 2m_Bm_{D^{(*)}} \right)$.
Exclusive $|V_{cb}|$ II

- $F(\omega)$ is the form-factor

$$\frac{d\Gamma(B \rightarrow D^* \ell \nu)}{dw} = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} K(\omega) F(\omega)^2$$

- $K(\omega)$ is the phase space factor, which goes to zero as $\omega \rightarrow$, so data must be extrapolated. There are theoretical models for the shape of $F(\omega)$. All that’s necessary is the lifetime, the value of the branching fraction at $F(1)$, which determines $(F(1)|V_{cb}|)^2$, & the theoretically determined corrections to $F(1)$ from 1
Predictions of $F(1)$

- Lattice (FNAL/MILC): $0.906 \pm 0.004 \pm 0.012$
- QCD sum rules: $0.86 \pm 0.02$

$|V_{cb}| \times 10^3 = 39.04 \pm 0.49_{\text{exp}} \pm 0.53_{\text{QCD}} \pm 0.19_{\text{QED}}$ (Lattice)

$= 41.6 \pm 0.6_{\text{exp}} \pm 1.9_{\text{thy}}$ (Sum rules)
Inclusive $|V_{cb}|$

- Here assume that the ensemble of exclusive $b \rightarrow c$ decays, $B \rightarrow D \ell \nu, D^* \ell \nu, D^{**} \ell \nu,...$ can be approximated by a continuum, called “duality”. The model is called the Heavy Quark Expansion (HQE).

- The decay rate is related to $|V_{cb}|$ as

$$\Gamma(B \rightarrow X_c \ell \bar{\nu}) = \frac{G_F^2 m_c^5 |V_{cb}|^2}{192\pi^3} \left( f(\rho) + k(\rho) \frac{\mu_\pi^2}{2 m_b^2} + g(\rho) \frac{\mu_G^2}{2 m_b^2} \right. \left. + d(\rho) \frac{\rho_D^3}{m_b^3} + l(\rho) \frac{\rho_{LS}^3}{m_b^3} + \mathcal{O}(m_b^{-4}) \right),$$

where $\rho = m_c^2 / m_b^2$, and $\mu_\pi^2, \mu_G^2, \rho_D$ and $\rho_{LS}$ are non-perturbative matrix elements of local operators.

- We will not go into the details here see arXiv:0902.3743
Inclusive $|V_{cb}|$

- Latest result: $|V_{cb}| \times 10^3 = 41.94 \pm 0.43_{\text{fit}} \pm 0.59_{\text{thy}}$
  \[= 41.94 \pm 0.73\]
- Exclusive (Lattice) \[= 39.04 \pm 0.75\]
- Difference has $\chi^2 = 3.8$ for 1 dof, prob = 5%
- Could there be a problem here?
- $\Lambda_b/B^0$ lifetime ratio: HQE predicts that the lifetime ratio is almost equal, with $\Lambda_b$ being shorter by a few %.
**$\Lambda_b/B^0$ lifetime ratio**

- $\Lambda_b$ lifetime measurements were much lower.
- LHCb now finds

$$\frac{\tau_{\Lambda_b}}{\tau_{B^0}} = 0.974 \pm 0.006 \pm 0.004.$$  

- Consistent with HQE original prediction.

Credit Uraltsev

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**Experiment**
- LHCb (3/fb) (2014) [J/$\psi K^-$]
- LHCb (1/fb) (2014) [J/$\psi \Lambda$]
- LHCb (1/fb) (2013) [J/$\psi pK^-$]
- CMS (2012) [J/$\psi \Lambda$]
- ATLAS (2012) [J/$\psi \Lambda$]
- D0 (2012) [J/$\psi \Lambda$]
- CDF (2011) [J/$\psi \Lambda$]
- CDF (2010) [$\Lambda_c^+ \pi^-$]
- D0 (2007) [J/$\psi \Lambda$]
- DLPH (1999) [Semileptonic decay]
- ALEP (1998) [Semileptonic decay]
- OPAL (1998) [Semileptonic decay]
- CDF (1996) [Semileptonic decay]

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**Exclusive $|V_{ub}|$**

- No theory like HQET
- Must rely on Lattice & model calculations

### Exclusive decays

<table>
<thead>
<tr>
<th>Decay</th>
<th>HPQCD ($q^2 &gt; 16$) (HFAG) \cite{97,11}</th>
<th>Fermilab/MILC ($q^2 &gt; 16$) (HFAG) \cite{98,11}</th>
<th>Lattice, full $q^2$ range (HFAG) \cite{11}</th>
<th>LCSR ($q^2 &lt; 12$) (HFAG) \cite{100,11}</th>
<th>LCSR ($q^2 &lt; 16$) (HFAG) \cite{101,11}</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{B} \rightarrow \pi l\bar{\nu}_l$</td>
<td>$3.52 \pm 0.08^{0.61}_{0.40}$</td>
<td>$3.36 \pm 0.08^{0.37}_{0.31}$</td>
<td>$3.28 \pm 0.29$</td>
<td>$3.41 \pm 0.06^{+0.37}_{-0.32}$</td>
<td>$3.58 \pm 0.06^{+0.59}_{-0.40}$</td>
</tr>
</tbody>
</table>

See Ricciardi arXiv:1403.7750
**Exclusive $|V_{ub}|$**

- Use HQE. Here many final states possible

| Models  | Inclusive decays $(|V_{ub}| \times 10^3)$ | See Ricciardi arXiv:1403.7750 |
|---------|----------------------------------------|-------------------------------|
| Babar   | $4.28 \pm 0.24^{+0.18}_{-0.20}$        | $4.35 \pm 0.24^{+0.09}_{-0.10}$ |
| Belle   | $4.47 \pm 0.27^{+0.19}_{-0.21}$        | $4.54 \pm 0.27^{+0.10}_{-0.11}$ |
| HFAG    | $4.40 \pm 0.15^{+0.19}_{-0.21}$        | $4.39 \pm 0.15^{+0.12}_{-0.20}$ |

- So take e.g. exclusive $(3.28 \pm 0.29) \times 10^{-3}$
- & inclusive $(4.20 \pm 0.25) \times 10^{-3}$
- These are inconsistent!
- No resolution in sight

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Summary

Note

$\bar{\rho} = \rho (1 - \lambda^2 / 2)$

$\bar{\eta} = \eta (1 - \lambda^2 / 2)$

Bands are $\pm 2\sigma$
Neutral Meson Mixing

- Neutral heavy mesons can transform into their anti-particles via 2nd order weak interactions.
- Short distance transition rate depends on
  - Mass of intermediate $q_i$, the heavier the larger, favors mesons containing s & b, since t is allowed
  - CKM elements $V_{ij}$.

$D^0$, $B^0_d$, $B^0_s$ :

$\text{Prob}[D^0](t)$, $\text{Prob}[B^0_d](t)$, $\text{Prob}[B^0_s](t)$

New particles possible in the loop

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Mixing formalism

- Hamiltonian

\[ \mathcal{H} = M - \frac{i}{2} \Gamma = \left( \begin{array}{cc} M & M_{12} \\ M_{12}^* & M \end{array} \right) - \frac{i}{2} \left( \begin{array}{cc} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{array} \right) \]

- Schrödinger equation

\[ i \frac{d}{dt} \left( \begin{array}{c} |B^0(t)\rangle \\ \overline{B^0(t)} \end{array} \right) = \mathcal{H} \left( \begin{array}{c} |B^0(t)\rangle \\ \overline{B^0(t)} \end{array} \right) \]

- Diagonalizing

\[ \Delta m = m_{B_H} - m_{B_L} = 2 |M_{12}| \]

\[ \Delta \Gamma = \Gamma_L - \Gamma_H = 2 |\Gamma_{12}| \cos \phi \]

\[ \phi = \arg \left( -\frac{M_{12}}{\Gamma_{12}} \right) \]
B Mixing data

First seen by ARGUS

$\Delta m_d = 0.5156 \pm 0.0051 \text{ (stat)} \pm 0.0033 \text{ (syst)} \text{ ps}^{-1}$

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D^0-\bar{D}^0 Mixing

- D^{*+} \rightarrow \pi^+ D^0 provides an initial flavor tag

- “Wrong-sign” (WS) D^0 can appear via mixing or a rare decay that gives the same final state called doubly-Cabbibo suppressed decay (DCS), where DCS follow $\sim \exp(-t/\tau_{D^0})$. Mixing, however, depends on $t$ in a more complicated way.

- Define $R_D = \text{DCS/(Cabibbo favored)}$. Mixing is parameterized as $x'$ & $y'$, functions of $\Delta m$ & $\Delta \Gamma$.

- Measure Wrong-sign/Right-sign, $R(t) = (\text{WS/RS})$:

$$R(t) \approx R_D + \sqrt{R_D} \frac{y'}{\tau} \frac{t}{\tau} + \frac{x'^2 + y'^2}{4} \left( \frac{t}{\tau} \right)^2$$

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Charm mixing result

\[ \text{D}^{*+} \rightarrow \pi^+ \text{D}^0, \]
\[ \text{D}^0 \rightarrow K^- \pi^+ \text{ (RS)} \]
\[ \bar{\text{D}}^0 \rightarrow K^+ \pi^- \text{ (WS)} \]

No mixing excluded at 9.1\( \sigma \), systematic errors are included
\[ y' = (7.2 \pm 2.4)\% \]
\[ x'^2 = (-0.09 \pm 0.13)\% \]

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B mixing CKM constraints

- For $B^0$ mixing

\[ \frac{\Delta m}{\Gamma} = \frac{G_F^2}{6\pi^2} B_{B_d} f_B^2 m_B \tau_B |V_{tb}^* V_{td}|^2 m_t^2 F \left( \frac{m_t^2}{M_W^2} \right) \eta_{QCD}. \]

$B_B$ is a theoretical parameter, $f_B$, the meson decay constant is also estimated theoretically though in principle measuring $B^- \rightarrow \tau \nu$ would determine $|V_{ub}|^2 f_B^2$. $F$ is a known function & $\eta_{QCD} \approx 0.8$

- Similar Eq. for $B_s$ mixing. Errors cancel in

\[
\frac{x_d}{x_s} = \frac{B_B}{B_{B_s}} \frac{f_B^2}{f_{B_s}^2} \frac{m_B}{m_{B_s}} \frac{\tau_B}{\tau_{B_s}} \frac{|V_{tb}^* V_{td}|^2}{|V_{tb}^* V_{ts}|^2}
\]
We have
\[ |V^*_{tb} V_{td}|^2 = A\lambda^3 |(1 - \rho - i\eta)|^2 = A\lambda^3 (\rho - 1)^2 + \eta^2 \] and
\[ |V^*_{tb} V_{ts}|^2 = A\lambda^2 , \]

So the ratio gives a circle in the \((\bar{\rho}, \bar{\eta})\) plane centered at (1,0).

(Modulo small higher order corrections)
Sakharov conditions

- Big bang gave matter & anti-matter

- For the Universe to exist:
  1. Baryon # violation
  2. Departure from thermal equilibrium
  3. C & CP violation, where C is charge conjugation, e.g, $C|p> = \pm |p>$, & P is parity $P|\psi(r)> = \pm |\psi(-r)>$

  - 1. is satisfied as SM gives B violation at high T
  - 2. is satisfied from the EW phase transition
  - 3. C & CP are violated by weak interactions

- BUT amount of CPV is too small by $10^9$, so new sources need to be found
CP formalism

- Basic idea: two interfering amplitudes that ultimately involve the CKM parameter $\eta$.

\[ \Gamma(B \rightarrow f) = \left( |A| e^{i(s_A + w_A)} + |B| e^{i(s_B + w_B)} \right)^2 \]
\[ \Gamma(\bar{B} \rightarrow \bar{f}) = \left( |A| e^{i(s_A - w_A)} + |B| e^{i(s_B - w_B)} \right)^2 \]
\[ \Gamma(B \rightarrow f) - \Gamma(\bar{B} \rightarrow \bar{f}) = 2 |AB| \sin(s_A - s_B) \sin(w_A - w_B) \]

- Favorable if $A$ & $B$ are about the same size
- Resulting rate difference depends on both a strong & weak phase difference

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Consider specifically $|B^0>$, but this can be for any $P^0$: $K^0$, $B^0$, $B^0_s$, or $D^0$.

$\text{CP}|B^0> = |\overline{B}^0>$. So these are not CP eigenstates, but

$|B_1^0> = \frac{1}{\sqrt{2}} (|B^0> - |\overline{B}^0>)$ & $|B_2^0> = \frac{1}{\sqrt{2}} (|B^0> + |\overline{B}^0>)$ are with $\text{CP}|B_1^0> = |B_1^0>$ & $\text{CP}|B_2^0> = -|B_2^0>$

To allow for CPV define

$|B_L> = p|B^0> + q|\overline{B}^0>$, $|B_H> = p|B^0> - q|\overline{B}^0>$

where CP is violated if $|p/q| \neq 1$
Here we are interested in a final state that can be reached by either a $|P^0>$ or a $|\bar{P}^0>$.

Then we can utilize mixing to provide another Interfering amplitude.

$f$ can be a CP eigenstate, $CP|f_{CP}\rangle = \pm|f_{CP}\rangle$ but it doesn’t have to be.

Define $A = \langle f_{CP}|H|B^0\rangle$, $\overline{A} = \langle f_{CP}|H|\bar{B}^0\rangle$. If $\left|\frac{\overline{A}}{A}\right| \neq 1$ we have “direct” CPV, but all that is needed is for $\lambda = \frac{q}{p} \cdot \frac{\overline{A}}{A} \neq 1$ which can happen even if $\left|\frac{p}{q}\right| = \left|\frac{\overline{A}}{A}\right| = 1$. 

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The asymmetry is given by

\[
a_{f_{CP}} = \frac{\Gamma (B^0(t) \rightarrow f_{CP}) - \Gamma (\bar{B}^0(t) \rightarrow f_{CP})}{\Gamma (B^0(t) \rightarrow f_{CP}) + \Gamma (\bar{B}^0(t) \rightarrow f_{CP})}
\]

For \(|\lambda| = 1\), we have

\[
a_{f_{CP}} = -\text{Im} \lambda \sin (\Delta mt)
\]
CP mixing phase

- Depends on CKM elements in mixing or box diagram

For $B^0$

$$\frac{q}{p} = \frac{(V_{tb}V_{td}^*)^2}{|V_{tb}V_{td}|^2} = \frac{(1-\rho-i\eta)^2}{(1-\rho+i\eta)(1-\rho-i\eta)} = e^{-2i\beta}$$

- arg($p/q$) = $\beta$

For $B_s$

$$\frac{q}{p} = \frac{(V_{tb}V_{ts}^*)^2}{|V_{tb}V_{ts}|^2} = 1$$

- arg($p/q$) $\sim$ 0
CPV for $B^0$

- Need $p/q$ and $\bar{A}/A$. Choosing a suitable CP eigenstate forces $\bar{A}/A=1$. $p/q$ comes from mixing

$$\frac{q}{p} = \frac{(V_{tb}^* V_{td})^2}{|V_{tb} V_{td}|^2} = \frac{(1 - \rho - i\eta)^2}{(1 - \rho + i\eta)(1 - \rho - i\eta)} = e^{-2i\beta}$$

- $B^0$:

$$\text{Im} \frac{q}{p} = -\frac{2(1 - \rho)\eta}{(1 - \rho)^2 + \eta^2} = \sin(2\beta)$$

- This is SM
For charmonium final states (Belle)
Measurements of $\sin^2 \beta$

- Requires knowledge of $B$ flavor at birth – use info from the other $B$ in the event
- $\sin^2 \beta$ values
  - Belle: $0.667 \pm 0.023 \pm 0.012$
  - BaBar: $0.691 \pm 0.028 \pm 0.012$
  - World Average: $0.682 \pm 0.019$

$$\beta = \left( 21.5^{+0.8}_{-0.7} \right)^\circ \text{ or } \left( 68.5^{+0.7}_{-0.8} \right)^\circ$$
CPV in $B_s \rightarrow J/\psi X$

- For $f = J/\psi \phi$ or $J/\psi f_0$
  
  - Small CPV expected, good place for NP to appear. Non zero due to CKM effects of order $\lambda^4$ in $V_{ts}$
  
  - $J/\psi \phi$ not a CP eigenstate. Why? But can be used

\[
\phi_s^{SM} = -2 \beta_s = -2 \arg \left( \frac{V_{ts} V_{tb}^*}{V_{cs} V_{cb}^*} \right) = -2^\circ
\]
Consider

\[
a[f(t)] = \frac{\Gamma(M \rightarrow f) - \Gamma(\bar{M} \rightarrow f)}{\Gamma(M \rightarrow f) + \Gamma(\bar{M} \rightarrow f)}
\]

Define

\[
A_f \equiv A(M \rightarrow f), \quad \bar{A}_f \equiv A(\bar{M} \rightarrow f), \quad \lambda_f = \frac{p}{q} \frac{\bar{A}_f}{A_f}
\]

Only 1 \( A_f \) & \( \Delta \Gamma = 0 \)

\[
\Gamma(M \rightarrow f) = N_f |A_f|^2 e^{-\Gamma t} \left( 1 - \text{Im} \lambda_f \sin(\Delta M t) \right)
\]

Then

\[
a[f(t)] = -\text{Im} \lambda_f, \quad \text{&} \quad \lambda_f \text{ is a function of } V_{ij} \text{ in SM}
\]

For \( B^0, \Delta \Gamma \neq 0 \), but there can be multiple \( A_f \)

\[
\Gamma(M \rightarrow f) = N_f |A_f|^2 e^{-\Gamma t} \left( \frac{1 - |\lambda_f|^2}{2} \cos(\Delta M t) - \text{Im} \lambda_f \sin(\Delta M t) \right)
\]

If in addition \( \Delta \Gamma = 0 \), eg. \( B_s \)

\[
\Gamma(M \rightarrow f) = N_f |A_f|^2 e^{-\Gamma t} \left( \frac{1 + |\lambda_f|^2}{2} \cosh \frac{\Delta \Gamma t}{2} + \frac{1 - |\lambda_f|^2}{2} \cos(\Delta M t) - \text{Re} \lambda_f \sinh \frac{\Delta \Gamma t}{2} - \text{Im} \lambda_f \sin(\Delta M t) \right)
\]

See Nierste
### Transversity

\[
\frac{d^4\Gamma(B_s^0 \rightarrow J/\psi \phi)}{dt \, d\cos \theta \, d\varphi \, d\cos \psi} \equiv \frac{d^4\Gamma}{dt \, d\Omega} \propto \sum_{k=1}^{10} h_k(t)f_k(\Omega)
\]

<table>
<thead>
<tr>
<th>$k$</th>
<th>$h_k(t)$</th>
<th>$f_k(\theta, \psi, \varphi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$</td>
<td>A_0</td>
</tr>
<tr>
<td>2</td>
<td>$</td>
<td>A_\parallel(t)</td>
</tr>
<tr>
<td>3</td>
<td>$</td>
<td>A_\perp(t)</td>
</tr>
<tr>
<td>4</td>
<td>$\Re(A_\parallel(t)A_\perp(t))$</td>
<td>$- \sin^2 \psi \sin 2\theta \sin \phi$</td>
</tr>
<tr>
<td>5</td>
<td>$\Re(A_0(t)A_\parallel(t))$</td>
<td>$\frac{1}{2}\sqrt{2} \sin 2\psi \sin^2 \theta \sin 2\phi$</td>
</tr>
<tr>
<td>6</td>
<td>$\Im(A_0(t)A_\perp(t))$</td>
<td>$\frac{1}{2}\sqrt{2} \sin 2\psi \sin 2\theta \cos \phi$</td>
</tr>
<tr>
<td>7</td>
<td>$</td>
<td>A_s(t)</td>
</tr>
<tr>
<td>8</td>
<td>$\Re(A_s^*(t)A_\parallel(t))$</td>
<td>$\frac{1}{3}\sqrt{6} \sin \psi \sin^2 \theta \sin 2\phi$</td>
</tr>
<tr>
<td>9</td>
<td>$\Im(A_s^*(t)A_\perp(t))$</td>
<td>$\frac{1}{3}\sqrt{6} \sin \psi \sin 2\theta \cos \phi$</td>
</tr>
<tr>
<td>10</td>
<td>$\Re(A_s^*(t)A_0(t))$</td>
<td>$\frac{4}{3}\sqrt{3} \cos \psi(1 - \sin^2 \theta \cos^2 \phi)$</td>
</tr>
</tbody>
</table>

for S-wave under $\phi$ predicted by Stone & Zhang PRD 79, 074024 (2009)
Transversity II

\[
|A_0|^2(t) = |A_0|^2 e^{-\Gamma_s t} \left[ \cosh \left( \frac{\Delta \Gamma}{2} t \right) - \cos \phi_s \sinh \left( \frac{\Delta \Gamma}{2} t \right) + \sin \phi_s \sin(\Delta m t) \right],
\]

\[
|A_\parallel(t)|^2 = |A_\parallel|^2 e^{-\Gamma_s t} \left[ \cosh \left( \frac{\Delta \Gamma}{2} t \right) - \cos \phi_s \sinh \left( \frac{\Delta \Gamma}{2} t \right) + \sin \phi_s \sin(\Delta m t) \right],
\]

\[
|A_\perp(t)|^2 = |A_\perp|^2 e^{-\Gamma_s t} \left[ \cosh \left( \frac{\Delta \Gamma}{2} t \right) + \cos \phi_s \sinh \left( \frac{\Delta \Gamma}{2} t \right) - \sin \phi_s \sin(\Delta m t) \right],
\]

\[
\Im(A_\parallel^*(t) A_\perp(t)) = |A_\parallel||A_\perp| e^{-\Gamma_s t} \left[ -\cos(\delta_\perp - \delta_\parallel) \sin \phi_s \sinh \left( \frac{\Delta \Gamma}{2} t \right) 
- \cos(\delta_\perp - \delta_\parallel) \cos \phi_s \sin(\Delta m t) + \sin(\delta_\perp - \delta_\parallel) \cos(\Delta m t) \right],
\]

\[
\Re(A_0^*(t) A_\parallel(t)) = |A_0||A_\parallel| e^{-\Gamma_s t} \cos(\delta_\parallel - \delta_0) \left[ \cosh \left( \frac{\Delta \Gamma}{2} t \right) - \cos \phi_s \sinh \left( \frac{\Delta \Gamma}{2} t \right) 
+ \sin \phi_s \sin(\Delta m t) \right],
\]

\[
\Im(A_0^*(t) A_\perp(t)) = |A_0||A_\perp| e^{-\Gamma_s t} \left[ -\cos(\delta_\perp - \delta_0) \sin \phi_s \sinh \left( \frac{\Delta \Gamma}{2} t \right) 
- \cos(\delta_\perp - \delta_0) \cos \phi_s \sin(\Delta m t) + \sin(\delta_\perp - \delta_0) \cos(\Delta m t) \right],
\]

\[
|A_s(t)|^2 = |A_s|^2 e^{-\Gamma_s t} \left[ \cosh \left( \frac{\Delta \Gamma}{2} t \right) + \cos \phi_s \sinh \left( \frac{\Delta \Gamma}{2} t \right) - \sin \phi_s \sin(\Delta m t) \right],
\]

\[
\Re(A_s^*(t) A_\parallel(t)) = |A_s||A_\parallel| e^{-\Gamma_s t} \left[ -\sin(\delta_\parallel - \delta_s) \sin \phi_s \sinh \left( \frac{\Delta \Gamma}{2} t \right) 
- \sin(\delta_\parallel - \delta_s) \cos \phi_s \sin(\Delta m t)
+ \cos(\delta_\parallel - \delta_s) \cos(\Delta m t) \right],
\]

\[
\Im(A_s^*(t) A_\perp(t)) = |A_s||A_\perp| e^{-\Gamma_s t} \sin(\delta_\perp - \delta_s) \left[ \cosh \left( \frac{\Delta \Gamma}{2} t \right) + \cos \phi_s \sinh \left( \frac{\Delta \Gamma}{2} t \right) 
- \sin \phi_s \sin(\Delta m t) \right],
\]

\[
\Re(A_s^*(t) A_0(t)) = |A_s||A_0| e^{-\Gamma_s t} \left[ -\sin(\delta_0 - \delta_s) \sin \phi_s \sinh \left( \frac{\Delta \Gamma}{2} t \right) 
- \sin(\delta_0 - \delta_s) \cos \phi_s \sin(\Delta m t) + \cos(\delta_0 - \delta_s) \cos(\Delta m t) \right].
\]
$\phi_s$ from $B_s \rightarrow J/\psi \pi^+ \pi^-$

- Reconstructed $\pi^+ \pi^-$ mass spectrum
- In region between arrows, measured to be >97.7%
- CP-odd @95% cl

$$\alpha[f(t)] = 2 \sin \phi_s \sin(\Delta M t)$$

- $\phi_s = -0.019^{+0.173+0.004}_{-0.174-0.003}$ rad (1/fb)
- (uncertainty for 3/fb~0.070 rad)

Fermilab Academic Lectures, May, 2014
Combining LHCb results:

\[ \phi_s = 0.01 \pm 0.07 \pm 0.01 \text{ rad} \]
\[ \Gamma_s = 0.661 \pm 0.004 \pm 0.006 \text{ ps}^{-1} \]
\[ \Delta \Gamma_s = 0.106 \pm 0.011 \pm 0.007 \text{ ps}^{-1} \]