B_d and B_s mixing at LHCb

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The performances of the re-optimized LHCb detector is studied. In the first part, the reconstruction and selection efficiencies are given for different channels which will be used to estimate the sensitivities to the mixing parameters $\Delta m$ and $\phi$. The high statistics of LHCb will allow to reduce the uncertainty on $\Delta m_d$ and $\phi_d$, and determine $\Delta m_s$ and $\phi_s$.

1 Theoretical overview

The goal of the LHCb experiment is to perform systematic measurements of CP violating processes and rare decays in the $B$-meson system with unprecedented precision. It will exploit the $500 \mu b$ beauty production cross-section at the 14 $TeV$ proton-proton collisions of the LHC. Measuring CP violation in many different decay modes of $B^0 D$ and $B^0 S$ mesons and comparing the results to predictions from the Standard Model will also allow a search for new physics.

1.1 CP violation formalism

In the framework of the Standard Model (SM), CP-violation effects originate from the charged-current interaction of quarks. Its description is based on the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1, 2], $V_{CKM}$, which connects the electroweak eigenstates ($d'$, $s'$, $b'$) to the corresponding mass eigenstates ($d$, $s$, $b$).

1.1.1 The “unitarity triangles”

The unitarity of the CKM matrix leads to a set of six orthogonality relations that can be represented as six triangles in the complex plane, all having the same area. Only two of them have three sides of comparable magnitude, each of order $O(\lambda^3)$, where $\lambda = \sin \theta_{\text{Cabibbo}}$. For the remaining triangles, one side is suppressed with respect to the others by factors of $O(\lambda^2)$ or $O(\lambda^4)$ [5]. The angles of the two unitarity triangles relate to the weak phases that one wishes to measure in order to test the CKM picture.

The $b \to c \tau \bar{\nu}$ transitions are used to determine the CP-violating phases $\phi_d$ (equal to the angle $2\beta$ of the unitarity triangle) and $\phi_s$ (equal to $-2\delta\gamma$). These phases are given by $\phi_q = 2\arg[V_{\bar{c}q}^{*}V_{cb}]$, where $q$ stands for $d$ or $s$ [5]. The $b \to c \tau \bar{\nu}$ transitions to CP eigenstates are dominated by only one CKM phase, the tree decay phase $\phi_D = -\arg[V_{\bar{c}b}^{*}V_{cs}]$. Before decaying, the $B$ mesons can oscillate into each other; the mixing-induced CP-violation arises from a phase difference ($\phi_{CKM}$) between $\phi_q$ and $\phi_D$. As the latter phase is expected to be small with respect to the former,
the CKM phase can be written as \( q = d, s \):

\[
\phi_{CKM} = \phi_q - 2\phi_D \sim \phi_q \neq 0, \pi.
\]

1.1.2 New physics

The CKM phases for the \( B_d^0 \)-system, \( \phi_d \), is well measured: \( \sin \phi_d \approx 0.7 \) [7]. It is not the case for \( \phi_s \), nor for \( \Delta m_s \), which can not be measured at the present \( B \)-factories. The SM prediction for this phase is: \( \sin \phi_s^{SM} \approx -0.04 \) radians [8].

The \( B_s^0 \)-system is often considered a “highway” towards new physics. For example, in the SUSY model, the contribution to \( B_s^0 - \bar{B}_s^0 \) transitions (mainly induced by gluino exchanges) could result in dramatic differences from SM predictions, giving for example \( \sin \phi_s \sim -1 \) [9]. In the up-type singlet models (with a quark mixing matrix or the dimension \( (3 + n_u) \times 3 \), where \( n_u \) is the number of additional up quarks) the mixing phase is given by \( \sin \phi_s \sim \lambda \sim 0.22 \) [10].

1.2 \( B - \bar{B} \) asymmetries

The time dependent asymmetries depend on the final state \( f(\bar{f}) \) of the \( B(\bar{B}) \)-meson decay. Henceforth the symbols \( B \) and \( \bar{B} \) will be used to denote the particle and antiparticle states for both \( B_d^0 \) and \( B_s^0 \) systems.

1.2.1 Case of \( f \) being a CP eigenstate

If \( f \) is a CP eigenstate, one has \( f = \bar{f} \), and four decay amplitudes reduce to two. Neglecting any direct CP violation in the mixing box-diagram, we define the time dependent CP asymmetry as:

\[
A_{CP}^f(t) = \frac{\Gamma_{\bar{f}} - \Gamma_{-f}(t)}{\Gamma_{\bar{f}} + \Gamma_{-f}(t)} = \frac{(1 - |\lambda_f|^2) \cos \Delta m t - 2 \text{Im}(\lambda_f) \sin \Delta m t}{(1 + |\lambda_f|^2) \cosh \frac{\Delta \Gamma}{2} t - 2 \text{Re}(\lambda_f) \sinh \frac{\Delta \Gamma}{2} t},
\]

where \( \Delta m = m_{B^0} - m_{B^0_L} \), \( \Delta \Gamma = \Gamma_{\bar{f}} - \Gamma_{-f} \), and \( \lambda_f = \frac{\lambda_f}{|\lambda_f|^2} \), with the amplitudes defined as \( A_f = A(B \to f) \) and \( \bar{A}_f = A(\bar{B} \to \bar{f}) \), and where \( \lambda \) stands for Heavy and L for Light. Introducing the quantities \( A_{d|s} = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \), \( A_{mx}^{d|s} = \frac{2 \text{Im}(\lambda_f)}{1 + |\lambda_f|^2} \) and \( A_{\Delta} = \frac{2 \text{Re}(\lambda_f)}{1 + |\lambda_f|^2} \). The time dependent CP asymmetry (1) can then be re-written as:

\[
A_{CP}^f(t) = \frac{A_{d|s}^{d|s} \cos \Delta m t + A_{mx}^{d|s} \sin \Delta m t}{\cosh \frac{\Delta \Gamma}{2} t - A_{\Delta} \sinh \frac{\Delta \Gamma}{2} t}.
\]

The quantities \( A_{d|s}^{d|s} \) and \( A_{mx}^{d|s} \) parameterize direct and mixing-induced CP violation respectively. The relative decay width difference \( \frac{\Delta \Gamma}{\Gamma} \) is expected to be of the order of 10% for the \( B_d^0 \) meson, while it is negligible for the \( B_s^0 \) meson [8].
1.2.2 Case of $f$ being a flavour-specific final state

If $f$ is a specific final state and $f \neq \bar{f}$, one can write $\lambda_f = \xi = 0$, since only the $B$ has instantaneous access to the decay channel $f$, whereas $\bar{B}$ only decays to $\bar{f}$. In this case, one can define the two decay asymmetries:

$$A_f^{CP}(t) = \frac{\Gamma_{\bar{B} \to f}(t) - \Gamma_{B \to f}(t)}{\Gamma_{\bar{B} \to f}(t) + \Gamma_{B \to f}(t)} = -\frac{\cos \Delta m t}{\cosh \frac{\Delta \Gamma}{2} t}.$$  

$$A_{\bar{f}}^{CP}(t) = \frac{\Gamma_{\bar{B} \to \bar{f}}(t) - \Gamma_{B \to \bar{f}}(t)}{\Gamma_{\bar{B} \to \bar{f}}(t) + \Gamma_{B \to \bar{f}}(t)} = \frac{\cos \Delta m t}{\cosh \frac{\Delta \Gamma}{2} t}.$$  

2 LHCb simulation

Estimations of the LHCb performance to the physics parameters, including consideration of trigger and tagging efficiencies, have been performed with a full Monte Carlo (MC) simulation. Pythia 6.2 was used at a center of mass energy of $\sqrt{s} = 14$ TeV for the generation of minimum bias proton-proton collisions; pile-up and spill-over effects were also included [6].

2.1 Reconstruction

Good mixing parameter sensitivity requires a precise reconstruction of a number of different decay channels, a high signal efficiency, and an efficient rejection of the combinatorial background as described in [6]. In order to have an efficient $B$ trigger and to precisely measure the time-dependent decays, good primary and decay vertex resolutions are essential; LHCb has primary- and secondary-vertex resolutions along the beam (z) axis of $\sigma_z(\text{prim. vert.}) \sim 45$ $\mu$m and $\sigma_z(\text{decay vert.}) \sim 100 - 200$ $\mu$m, respectively (channel dependent). It is also important to perform a background-free selection of the signal final states in order to reconstruct a clean $B$ mass distribution. Typical LHCb $B^0$ mass resolutions are: $\sim 15$ MeV/$c^2$ for charged particles final states, e.g. $B^0 \to J\psi \phi$, and $\sim 30$ MeV/$c^2$ for photons and charged particle final states, e.g. $B^0 \to J\psi \eta$.

To resolve fast $B^0_s - \bar{B}^0_s$ oscillations ($O(20 \text{ ps}^{-1})$), the $B$-meson proper time resolution has to be precise; LHCb has a resolution better than 50 $fs$ for all decays.

2.2 Flavour Tagging

Flavour tagging, i.e. the method of determining of whether the decaying $B(\bar{B})$ meson is particle or antiparticle, is necessary to study any decay which has a CP asymmetry or undergoes flavour oscillation. The statistical uncertainty on the measured CP asymmetries is directly related to the effective tagging efficiency $\epsilon_{eff}$, also known as "$\epsilon D^{2n}$", defined as

$$\epsilon_{eff} = \epsilon_{tag}(1 - 2\omega)^2,$$
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where $\varepsilon_{\text{tot}}$ is the tagging efficiency and $\omega$ is the wrong tag fraction. This uncertainty affects the statistical and the systematical precision, and the measured asymmetries are also affected, leading to:

$$A_{f}^{\text{obs}} = (1 - 2\omega)A_{f}^{\text{th}},$$

where $A_{f}^{\text{obs}}$ and $A_{f}^{\text{th}}$ are the measured and physical decay CP asymmetries, respectively.

The current LHCb tagging strategy is based on the opposite side lepton tag ($b \rightarrow l$), the opposite side kaon tag ($b \rightarrow c \rightarrow s$), the same side kaon tag (for $B_{s}^{0}$ only) and the opposite $B$ vertex charge tag. With this set of tagging methods, we can obtain an effective efficiency of $\varepsilon_{\text{eff}} \sim 4\%$ for the $B_{d}^{0}$ channels, and $\varepsilon_{\text{eff}} \sim 6\%$ for the $B_{s}^{0}$ channels [6].

2.3 Full MC results

The full MC simulation results, obtained for the decay channels reported in this paper are summarized in table 1. Here $\varepsilon_{\text{tot}}$ is the total signal efficiency (including acceptance, trigger and reconstruction efficiencies, but excluding tagging) and $B/S$ the background to signal ratio. As there are less than 10 background events selected for the three last decays in the table, estimates are quoted as 90\% CL upper limits.

<table>
<thead>
<tr>
<th>Decay channel</th>
<th>$\omega$ (%)</th>
<th>$\varepsilon_{\text{eff}}$ (%)</th>
<th>$\varepsilon_{\text{tot}}$ (%)</th>
<th>Yield (10^3/$fb$)</th>
<th>$B/S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{d}^{0} \rightarrow J/\psi K^{\pm}$</td>
<td>34.4</td>
<td>4.1</td>
<td>1.5</td>
<td>670</td>
<td>0.17</td>
</tr>
<tr>
<td>$B_{d}^{0} \rightarrow J/\psi K_{s}^{0}$</td>
<td>36.7</td>
<td>3.2</td>
<td>1.4</td>
<td>216</td>
<td>0.67</td>
</tr>
<tr>
<td>$B_{s}^{0} \rightarrow D_{s}^{+} \pi^{+}$</td>
<td>30.0</td>
<td>8.7</td>
<td>0.34</td>
<td>80</td>
<td>0.32</td>
</tr>
<tr>
<td>$B_{s}^{0} \rightarrow J/\psi \phi$</td>
<td>33.4</td>
<td>5.5</td>
<td>1.7</td>
<td>100</td>
<td>&lt; 0.3</td>
</tr>
<tr>
<td>$B_{s}^{0} \rightarrow J/\psi \eta$</td>
<td>30.0</td>
<td>4.8</td>
<td>0.46</td>
<td>7</td>
<td>&lt; 1.6</td>
</tr>
<tr>
<td>$B_{s}^{0} \rightarrow \eta_{c} \phi$</td>
<td>30.0</td>
<td>4.8</td>
<td>0.08</td>
<td>3</td>
<td>&lt; 0.8</td>
</tr>
</tbody>
</table>

Table 1. Tagging efficiencies, annual yields and background to signal ratios obtained from the full MC simulation for the relevant $B$ decay channels.

3 Sensitivity studies

The sensitivity of LHCb to selected CP observables have been assessed using “toy” Monte Carlo programs. These programs generate event samples under certain assumptions of the physics parameters to be measured and with the statistics expected at LHCb. The characteristics of the samples (signal resolution, efficiency, purity, etc.) are taken from the studies with fully-simulated events described in [6]. Systematic effects will be monitored from the data whenever this is possible (for example using control channels which have no expected CP violation to determine the asymmetry between $b$-hadron and $\bar{b}$-hadron production).
3.1 Sensitivity to $\Delta m_d$ using $B_d^0 \to J/\Psi K^{*0}$

This channel is not expected to exhibit any CP violation effects. It is therefore considered to be a good control channel to study systematic errors and to check LHCb tagging methods. The observable flavour asymmetry is:

$$A^{\text{obs}}(t) = (1 - 2\omega) \cdot \cos (\Delta m_d t)$$

The $B_d^0$ oscillation fit is shown in figure 1 corresponding to one year of LHCb data taking. The fitted value of the wrong tag fraction, $\omega = (36.5 \pm 1.0)\%$, will be used in the sensitivity studies.

3.2 Sensitivity to $\sin \phi_d$ using $B_d^0 \to J/\Psi K^{\pm}$

Besides being an interesting measurement in its own right, this channel allows a precise determination of $\phi_d$ on which other parameters, measured in other channels, can depend. Hence its measurement uncertainty can affect the sensitivity of LHCb to the other angles. This decay is also theoretically the cleanest way to measure $\phi_d$. Its asymmetry is given by equation 2:

$$A^{\text{obs}}(t) = (1 - 2\omega) \cdot \sin \phi_d \cdot \cos (\Delta m_d t).$$

Figure 2 shows the expected time dependent asymmetry that can be expected after one year of LHCb running. The sensitivity to $\sin \phi_d$ is $\sigma_{LHCb}(\sin \phi_d) = 0.022$. The world average combined measurement in 2007, at the start of the LHC, is predicted to be $\sigma_{2007}(\sin \phi_d) \sim 0.02$. The contribution of LHCb to this well-known asymmetry will be to add more statistics and, comparing to other channels, to indicate if new physics might be present through penguin diagrams.

3.3 Sensitivity to $\Delta m_s$ using $B_s^0 \to D_s^- \pi^+$

This decay allows the extraction of the parameters $\Delta m_s$, $\Delta \Gamma_s$ and $\omega$. As there is no CP-asymmetry, this decay is used as a control channel for studies measuring
The decay asymmetry follows formula 3 and is scaled with the time dependent efficiency, determined from MC study, and shown in figure 3.

$$A^{obs}(t) = -(1 - 2\omega) \cdot \frac{\cos (\Delta m_s t)}{\cosh (\frac{\Delta \Gamma}{2} t)}.$$  \hspace{1cm} (3)

The time-dependent fit to the $B_s^0 \rightarrow D_s^- \pi^+$ oscillations is shown in figure 4.

The sensitivity on $\Delta m_s$ after one year of LHCb running is $\sigma(\Delta m_s) = 0.013 \text{ ps}^{-1}$ for $\Delta m_s = 25 \text{ ps}^{-1}$. We expect to reach a sensitivity of more than 5$\sigma$ for $\Delta m_s < 68 \text{ ps}^{-1}$, which is well beyond the SM predictions [8].
3.4 Sensitivity to $\sin \phi_s$ using $B^0_s \rightarrow J/\Psi \phi$

The $B^0_s \rightarrow J/\Psi \phi$ channel is the $B^0_s$ counterpart of $B^0 \rightarrow J/\Psi K^0_s$, but in the former case, the final state is an admixture of CP eigenstates ($\eta_f$):
- CP-even configuration: $\eta_f = 1$ and the final state is $f = 0, \parallel$,
- CP-odd configuration: $\eta_f = -1$ and the final state is $f = \perp$.

To take these three distributions into account, linear polarization amplitudes are introduced: $A_f(t)$ for $f = 0, \parallel, \perp$ and the fraction of CP-odd is defined as $R_T \equiv |A_\perp(0)|^2 / \sum_f |A_f(0)|^2 \sim 20\%$. The distribution of the transversity angle $\theta_{tr}$ (defined as the angle between the positive charged lepton direction and the $\phi$ decay plane) allows us to distinguish between the different CP eigenstates:

$$\frac{d \Gamma(t)}{d \cos(\theta_{tr})} \propto \left[ |A_0(t)|^2 + |A_\parallel(t)|^2 \right] \frac{3}{8} (1 + \cos^2 \theta_{tr}) + |A_\perp(t)|^2 \frac{3}{4} \sin^2 \theta_{tr}.$$ 

A lifetime error is implemented event by event in the fast simulation in such a way that the experimental uncertainty is assigned to each generated event, based on a full MC simulation.

The physics parameters of the time-dependent CP asymmetry (for one given angular final state),

$$A^{obs}(t) = -(1 - 2\omega) \cdot \frac{\eta_f \sin \phi_s \sin (\Delta m_s t)}{\cosh (\Delta \Gamma_s t) - \eta_f \cos \phi_s \sinh (\Delta \Gamma_s t)},$$

are extracted using an maximum likelihood fit to the proper time, the mass distribution and the transversity angle. The fit is also simultaneously maximized with the control sample $B^0 \rightarrow D^- \pi^+$. The likelihood used is the product of a mass likelihood, a Gaussian for the signal, an exponential for the background, a decay rates likelihood including the resolution effects, and an angular contribution likelihood. Thousand toy experiments, each corresponding to one year of LHCb running, are performed.

The fit procedure is carried out as follows:

- The mass distributions are fitted and the per-event signal probability determined, based on the reconstructed mass.
- The sidebands are then used to determine the background parameters.
- In the signal window, the physics parameters are then fitted.

The expected sensitivities to $\Delta \Gamma_s / \Gamma_s$ and $\phi_s$ are:

<table>
<thead>
<tr>
<th>Sensitivity (one year)</th>
<th>$\sigma(\Delta \Gamma_s / \Gamma_s)$</th>
<th>$\sigma(\phi_s)$ [rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0_s \rightarrow J/\Psi \phi$</td>
<td>0.018</td>
<td>0.06</td>
</tr>
<tr>
<td>$B^0_s \rightarrow J/\Psi \eta$</td>
<td>$\sim 0.025$</td>
<td>$\sim 0.1$</td>
</tr>
<tr>
<td>$B^0_s \rightarrow \eta_c \phi$</td>
<td>$\sim 0.025$</td>
<td>$\sim 0.1$</td>
</tr>
<tr>
<td>Combined sensitivity</td>
<td>$\sim 0.05$</td>
<td></td>
</tr>
</tbody>
</table>
Two other channels are added to the one of interest: $B_s^0 \rightarrow J/\Psi \eta$ and $B_s^0 \rightarrow \eta_c \phi$. These decay to pure CP-eigenstates (CP-even), which do not need angular analysis. The results given for these two channels are preliminary. The statistical sensitivity to $\phi_s$ after five years of LHCb data taking has been estimated to $\sigma(\phi_s) \sim 0.02$ rad, with $\phi_s \sim -0.04$ rad expected in the SM.

4 Summary

We have presented methods to extract the CP-phases and $\Delta m_s$ at LHCb using different decay channels. The sensitivities after one year to the parameters of interest are: $\Delta m_s > 5 \sigma$ for a $\Delta m_s < 68 \text{ps}^{-1}$, $\sigma_{LHCb}(\sin \phi_d) \sim 0.022$ and $\sigma_{LHCb}(\sin \phi_s) \sim 0.05$. The LHCb contribution to the determination of these parameters after one year of running will be to:

- reduce the uncertainties on $\Delta m_d$ and $\sin \phi_d$,
- measure very precisely $\Delta m_s$, and
- determine $\sin \phi_s$ to $2 \sigma$ within five years if it is within the SM prediction, or to $4 \sigma$ within 1 year if $\sin \phi_s \sim \lambda$.

The $B_s^0 - \bar{B}_s^0$ system is a prime topic for the discovery of new physics.

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