$\alpha_{\text{CKM}}$ from $B \rightarrow \pi^+\pi^-\pi^0$
- LHCb sensitivity -

O. Deschamps, A. Robert
*LPC Clermont-Ferrand*
On behalf of the LHCb collaboration
**$\alpha_{CKM}$ from $B \to \pi^+\pi^-\pi^0$**

Thanks to the interferences between the $\rho^+\pi^-\rho^0\pi^0\to \pi^-\pi^0\pi^+$ transitions, we can simultaneously extract $\alpha_{CKM}$ with amplitudes and strong phases. [Snyder, Quinn, 1993]

The time dependence of the tagged Dalitz plot distributions provides all the required information: $f(s^+,s^-,t,B_{tag})$

\[ M^\pm(s^+,s^-,t) = e^{\frac{i\eta}{2}} \left\{ \cos\left(\frac{\Delta m}{2}t\right)A^\pm(s^+,s^-) + i\left(\frac{q}{p}\right)^{\pm\mp} \sin\left(\frac{\Delta m}{2}t\right)A^\mp(s^+,s^-) \right\} \]
Maximize a Likelihood with $(9 \text{ parameters } \vec{\alpha} + \text{ backgrounds fractions } \vec{r})$

**Theoretical ingredients**

$$A_{3\pi} = f^+A^{+-} + f^-A^{-+} + f^0A^{00}$$

with $A^{ij} = e^{-i\vec{\alpha} \cdot \vec{r}^{ij}} + P^{ij}$

and $(P^{+-} + P^{-+}) = -2P^{00}$

$\vec{\alpha} = (\alpha, T^{+-}, \phi^{+-}, T^{00}, \phi^{00}, P^{+-}, \delta^{+-}, P^{00}, \delta^{00})$

**Phenomenological ingredients**

The $\rho$ line-shape

$$f^{\pm 0} \propto \left( f^{\pm 0}_{\rho^{770}} + \beta f^{\pm 0}_{\rho^{1450}} + \gamma f^{\pm 0}_{\rho^{1700}} \right) \times Y^{01}(\cos \theta^{\pm 0}(s^+, s^-))$$

Use Kuhn-Santa Maria parametrisation including first radial excitations of the $\rho$

$$L(\vec{\alpha}, \vec{r}) = \prod_k \left( 1 - r \right) \xi^{3\pi}_\alpha(s^+_k, s^-_k, t_k) \sum_{b=B,\bar{B}} \omega^\text{tag}_b M^3\pi_b(s^+_k, s^-_k, t_k, \vec{\alpha})^2 + \sum_{b=\text{bkg}} r^\text{bkg}_b \sum_{k=\text{bkg}} b^\text{bkg}_k \bigotimes G(\sigma^+, \sigma^-, \sigma_t)$$

**Event Yield**

**Experimental acceptances**

**Experimental (mis)tagging**

$\text{tag} = +1/0/-1$

**Background contamination**

**Experimental resolutions**

**Experimental ingredients**
Next generation B factory

- $\sqrt{s} = 14$ TeV
- $L = 2.10^{32}$ s$^{-1}$
- $\sigma_{bb} \sim 500 \mu$b

- reconstruct a large variety of (rare) B decay modes including those decays with prompt $\gamma, \pi^0, \ldots$

One B$\rightarrow \pi^+\pi^-\pi^0$ decay every 2 seconds

Experimental challenge:

- Reconstruct B decay in a high multiplicity environment
Resolved $\pi^0$: neutral pion reconstructed from a pair of isolated photons

- mass resolution $\sim 10$ MeV/c$^2$

Merged $\pi^0$: neutral pion reconstructed from a single large cluster using a dedicated algorithm

- (for high energy $\pi^0$, the showers from photons pair merge into a single large cluster)

- mass resolution $\sim 15$ MeV/c$^2$

$\pi^0$ reconstruction efficiency in $B\rightarrow 3\pi$ events

- $\langle \varepsilon \rangle = 53\%$
  - 33% from resolved
  - + 20% from merged
Multivariate selection based on:

- Particle identification
  Charged pion Id, neutral $\pi^0$ clusters, ...
- Kinematical criteria
  Transverse momenta, ...
- Vertexing criteria
  Impact parameters, vertex isolation, ...

Combined PDF

$$X_{PDF} = \frac{\ln(B) - \ln(S)}{\ln(B) + \ln(S)} \begin{cases} S(\bar{x}) = \prod_i s_i(x_i) \\ B(\bar{x}) = \prod_i b_i(x_i) \end{cases}$$

- Mass window around B mass: $\pm 200\text{MeV}/c^2$
- Central region of Dalitz plot removed
**Experimental ingredients: $B \rightarrow \rho \pi$ selection**

- **Selection efficiency**

<table>
<thead>
<tr>
<th>$\varepsilon_{\text{det-rec}}$</th>
<th>$\varepsilon_{\text{sel}}$</th>
<th>$\varepsilon_{\text{trig}}$</th>
<th>$\varepsilon_{\text{tot}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04</td>
<td>0.035</td>
<td>0.5</td>
<td>$7 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

- **Expected annual yield (2 fb$^{-1}$/year):**

  $N_{3\pi} = 14 \times 10^3$ evt/2 fb$^{-1}$

  (50% with merged $\pi^0$)

- **Inclusive $b\bar{b}$ background reduced by a factor $5 \times 10^7$**

  $\rightarrow$ Expect: $B/S \sim 0.80$  (B/S < 3 @ 90% CL)

  $b\bar{b}$ contamination is mainly due to combinatorial fragments from charmed $B$ decays.
Experimental ingredients: acceptance

- **Acceptance in Dalitz plane**
  - Produced vs. selected events
  - Lower corner of Dalitz plot (soft $\pi^0$ region) depopulated due to the large $\pi^0$ energy required in the selection

- **Proper time acceptance**
  - Region of low lifetime depopulated due to the large impact parameters required in the selection
Experimental ingredients: tagging & background

- **Expected resolutions**:
  - Resolutions are dominated by Ecal energy resolution.
  - Expected resolutions:
    - $\sigma \sim 60 \text{ MeV}/c^2$
    - $\sigma \sim 50 \text{ fs}$
    - $\sigma \sim 30 \text{ MeV}/c^2$

- **Flavour tagging**:
  - Tagging efficiency $\varepsilon = 40 \pm 2 \%$
  - Wrong tag fraction $\omega = 31 \pm 2 \%$
  \[ \varepsilon_{\text{eff}} = \varepsilon (1 - 2\omega)^2 = 6 \pm 2 \% \]

NB: the untagged sample also enters in the global fit:
\[ \{\omega_{\text{tag}}\} = \begin{pmatrix} 1 - \omega & 1/2 \\ \omega & 1/2 \\ 1 - \omega \end{pmatrix} \]
Sensitivity to $\alpha_{CKM}$ : the method

- For a given set of theoretical parameters $\alpha_{\text{gen}}$, simulate $10^3$ experiments made of $10^4$ $B \rightarrow 3\pi$ decays each (almost the expected annual yield)

<table>
<thead>
<tr>
<th>$\alpha_{CKM}$</th>
<th>$T^+$</th>
<th>$\Phi^+$</th>
<th>$T^0$</th>
<th>$\Phi^0$</th>
<th>$P^+$</th>
<th>$\delta^+$</th>
<th>$P^-$</th>
<th>$\delta^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>77.35° &amp; 106.0°</td>
<td>0.47</td>
<td>0.00</td>
<td>0.14</td>
<td>0.00</td>
<td>-0.2</td>
<td>-0.5</td>
<td>0.15</td>
<td>2.0</td>
</tr>
</tbody>
</table>

- Simulate background contamination according to given B/S and $\alpha_{\text{gen}}$ fractions
- Simulate experimental effects (resolution, acceptance, wrong tag, ...)
- Maximize the likelihood to extract $\alpha_{\text{fit}}$ and the background fractions $\rho_{\text{fit}}$

In a first attempt, the experimental ingredients are assumed to be perfectly known → the same parametrisations are used in the toy simulation and in the likelihood ...
Sensitivity to $\alpha_{\text{CKM}}$ : experimental impact

- Experimental ingredients successively added (except background):

<table>
<thead>
<tr>
<th>Configuration ((\alpha_{\text{gen}}=77.35^\circ) - (N_{3\pi}=10^4) - no background)</th>
<th>(\langle \alpha \rangle^{\text{fit}} (^\circ))</th>
<th>(\sigma_\alpha (^\circ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal experiment (with (10^4) signal events)</td>
<td>77.4 ± 0.1</td>
<td>1.2</td>
</tr>
<tr>
<td>(\oplus) Proper time acceptance</td>
<td>77.4 ± 0.1</td>
<td>0.9</td>
</tr>
<tr>
<td>(\oplus) Tagging</td>
<td>77.7 ± 0.1</td>
<td>3.1</td>
</tr>
<tr>
<td>(\oplus) Proper time and Dalitz coordinates resolution</td>
<td>77.4 ± 0.1</td>
<td>3.1</td>
</tr>
<tr>
<td>(\oplus) Dalitz acceptance</td>
<td>77.5 ± 0.2</td>
<td>4.4</td>
</tr>
<tr>
<td>All combined effects</td>
<td>77.2 ± 0.2</td>
<td>4.4</td>
</tr>
</tbody>
</table>

- Main impact on \(\sigma_\alpha\) due to tagging and selection acceptance in the Dalitz plane.

- ➔ accurate knowledge of these experimental ingredients will be needed.
Sensitivity to $\alpha_{CKM}$: background impact

- Poor knowledge of background sources
  - Use generic model mixing Flat & Resonant contributions $\rightarrow$ 11D fit: $9 \alpha \oplus r = \{B/S, R/B\}$
  - Assume same acceptances & resolutions as for signal

- Result for B/S=0.8
  - $\alpha_{gen}=77.4^\circ \Rightarrow <\alpha_{fit}> = (77.8^{+7}_{-5})^\circ$
  - $\alpha_{gen}=106.0^\circ \Rightarrow <\alpha_{fit}> = (103.0^{+12}_{-5})^\circ$

- $\alpha_{CKM}$ resolution almost independent on R/B ratio

- Distribution of fit error
  - (symmetric) error below 10° for about 90% of the experiments.
  - Few unlucky experiments with error up to 20°
Background fractions nicely fitted

B/S & R/B resolution better than 2%

σα as a function of B/S

For B/S=3

a_{\text{gen}}=77.4^\circ \Rightarrow \quad < \sigma_\alpha > \sim ( +8 \, \, -8 )^\circ

a_{\text{gen}}=106.0^\circ \Rightarrow \quad < \sigma_\alpha > \sim ( +22 \, \, -7 )^\circ
Scan of $\alpha_{CKM}$

Fix $\alpha_{CKM}$ value and minimize $\chi^2 = -2\ln(\mathcal{L})$ with respect to $N-1 = 10$ parameters

The correct solution generally corresponds to a deep (if not deepest) minimum

Dangerous mirror solution at $\pi/2 - \alpha_{CKM}$

$\sim 10\text{-}15\%$ of experiments have an absolute minimum other than the correct (expected) one
Sensitivity to $\alpha^{\text{gen}}_{\text{CKM}}$ : the ($\rho, \eta$) plane

Translate $\Delta \chi^2 = -2\ln(L)$ into the ($\rho, \eta$) plane for a typical experiment

$\alpha^{\text{gen}} = 77.37^\circ$
Penguin strong phases (\(\alpha^{\text{gen}}=106^\circ\))
\[\sigma(\delta^{-+}) \sim ({}^{+20}_{-50})^\circ\]
\[\sigma(\delta^{+-}) \sim ({}^{+4}_{-25})^\circ\]

Tree strong phases (\(\alpha^{\text{gen}}=106^\circ\))
\[\sigma(\Phi^{-+}) \sim ({}^{+6}_{-10})^\circ\]
\[\sigma(\Phi^{00}) \sim ({}^{+26}_{-17})^\circ\]

\(R=|P/T|\) ratios (\(\sigma^{\text{gen}}=106^\circ\))
\[\sigma_{R^{-+}/R^{+-}} \sim ({}^{+50}_{-30})\%\]
\[\sigma_{R^{+-}/R^{-+}} \sim ({}^{+70}_{-10})\%\]

- Highly asymmetric distributions
- Almost independent of Flat/Resonant ratio
- Similar resolutions for \(\sigma^{\text{gen}}=77.35^\circ\)
Systematic bias due to misleading experimental or phenomenological ingredients in the Likelihood

- $\not L$ not accounting for $\rho/\omega$ mixing in signal
- $\not L$ not accounting for 10\% $K\pi\pi$ in bkg
- No experimental acceptance in fit
- No proper time acceptance in fit
- No Dalitz acceptance in fit
- Non-uniform wrong-tag - averaged in fit
- $\not L$ not accounting for $\rho_3$ contribution ($\kappa=0.2$)
- $\not L$ not accounting for $\rho'$ and $\rho''$ contribution

A poor description of $\rho$ line-shape or Dalitz acceptance leads to large bias on $\alpha_{\text{CKM}}$

→ An accurate knowledge of these inputs is required.
Conclusions

- With 2 fb⁻¹ LHCb may achieve $\sigma_a^{\text{stat}} \leq 10^°$ from $B \rightarrow \pi^+\pi^-\pi^0$

assuming an accurate control of experimental and phenomenological ingredients

- Large LHCb MC production will be available in coming weeks
  - more accurate B/S estimate
  - 2D acceptance, tagging, B/S will be implemented
  - Develop strategies to extract/validate experimental inputs from data

- To prevent any question on that: $B \rightarrow \rho\rho$ modes are being studied!

Very preliminary:

- $10^4$ $B^+ \rightarrow \rho^+\rho^0 /2fb^{-1}$ - B/S $\leq 1$
- few $10^3$ $B_d \rightarrow \rho^+\rho^- /2fb^{-1}$
- few $10^2$ $B_d \rightarrow \rho^0\rho^0 /2fb^{-1}$ (for BR=$10^{-6}$)

More info @ CKM2006!
$\alpha_{\text{CKM}}$ from $B \rightarrow \pi^+\pi^-\pi^0$ @ LHCb in 2010!

$\alpha_{\text{LHCb}} = 106^\circ$

Thanks to Muriel Pivk
• $\alpha$ from $B \rightarrow \pi^+\pi^-\pi^0$
• $\gamma$ from $B_s \rightarrow D_s K$
• $\Delta m_s$ & $\Delta m_d$
The lower corner of the Dalitz plot is highly depopulated due to selection.
Can we fully remove this region of interference between rho-bands?

- Depopulated lower corner (no background)
  
  \[ \langle \alpha_{CKM} \rangle = (77.2 \pm 4.4)° \]

- Fully removing the lower corner (no background)
  
  \[ \langle \alpha_{CKM} \rangle = (77.0 \pm 6.2)° \]

The lower Dalitz corner carries useful but not essential information. The upper half of the Dalitz is enough.
Sensitivity to $\alpha_{CKM}$: specific background

- **Contamination of** $B_d \rightarrow K^*\pi, K\rho, K^*\gamma$ **modes potentially dangerous**

  Estimate: $B/S \sim 6\%$ for $K^*\pi^0$ and $K^*\gamma$ B-decay

- **Toy simulation**
  
  $B/S=0.8$
  
  Flat:Resonant = 50:50 $\oplus$ 10% ($K^*\pi, K\rho$)

  $\alpha^{\text{gen}}=77.4^\circ \Rightarrow <\sigma_\alpha>=(-5^\circ, +10^\circ)$

  Assuming a perfect knowledge of the $\gamma_{CKM}$ value entering into the ($K^*\pi, K\rho$) amplitudes
Sensitivity to other parameters

- \( \alpha^{\text{gen}} = 106^\circ \)
- Flat: Resonant ratio = 40:60

<table>
<thead>
<tr>
<th></th>
<th>Toy</th>
<th>(&lt;\text{fit}&gt;)</th>
<th>(\sigma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T^-</td>
<td>47%</td>
<td>(49±3)%</td>
<td>9%</td>
</tr>
<tr>
<td>(\Phi^-)</td>
<td>0\°</td>
<td>(1.5±0.5)\°</td>
<td>+6\° -10\°</td>
</tr>
<tr>
<td>(\Phi^{00})</td>
<td>14%</td>
<td>(14±1)%</td>
<td>4%</td>
</tr>
<tr>
<td>(\Phi^{00})</td>
<td>0\°</td>
<td>(-1±1)\°</td>
<td>+26\° -17\°</td>
</tr>
<tr>
<td>P^-</td>
<td>-20%</td>
<td>(-11±6)%</td>
<td>+20% -2%</td>
</tr>
<tr>
<td>(\delta^-)</td>
<td>-28.6\°</td>
<td>(15±1)\°</td>
<td>+4\° -25\°</td>
</tr>
<tr>
<td>P^+</td>
<td>40%</td>
<td>(18±1)%</td>
<td>±6%</td>
</tr>
<tr>
<td>(\delta^+)</td>
<td>114.6\°</td>
<td>(135±5)\°</td>
<td>+20\° -50\°</td>
</tr>
</tbody>
</table>

![Tree Amplitudes](image1)

![Tree Phases](image2)

![Penguin Amplitudes](image3)

![Penguin Phases](image4)
Neutral pion reconstruction

(a) $\pi^0 \rightarrow \gamma \gamma$

(b) $\pi^0 \rightarrow \gamma (e^+e^-)$

$E_t(\pi^0)$ (GeV/c)

Invariant mass (MeV/c²)

E$\tau$($\pi^0$) (GeV/c)

Efficiency (%)