Extraction of $\gamma$ at LHCb with a Combined

$B_s \rightarrow D_{s}^{\pm}K^{\mp}$ and $B_d \rightarrow D^{\pm}\pi^{\mp}$ Analysis

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Abstract

A combined analysis of the CP violating observables in the channels $B_s \rightarrow D_{s}^{\pm}K^{\mp}$ and $B_d \rightarrow D^{\pm}\pi^{\mp}$ allows the CKM angle $\gamma$ to be determined in a manner which has certain advantages over the analysis of each channel separately. In particular, the combined analysis reduces greatly the number of ambiguous solutions which would otherwise be present. The potential of the method is illustrated using the expected performance of the LHCb experiment.
1 Introduction

The potential of $B_s \to D_s^\pm K^\mp$, $B_d \to D^*\pm\pi^\mp$ and $B_d \to D^{\pm}\pi^\mp$ decays for extracting the CKM angle $\gamma$ is well known [1]. An analysis based on any of these modes in isolation, however, suffers from the problem of discrete ambiguities, and in the case of $B_d$ decays, of difficulties posed by very small interference effects. This report explains how a combined analysis of, for example, $B_s \to D_s^+ K^-$ and $B_d \to D^{\pm}\pi^\mp$ under the assumption of U-spin symmetry, circumvents these problems. This strategy was proposed in [2] and is here explored in the context of the expected performance in these decays at LHCb.

2 Formalism

From measuring the two flavour tagged decay modes, $B_s \to D_s^+ K^-$, as a function of proper time, $t$, the CP asymmetry $\mathcal{A}_{CP}(D_s^+ K^-)(t)$ may be constructed:

$$\mathcal{A}_{CP}(D_s^+ K^-)(t) = \frac{B_s \to D_s^+ K^-(t) - \overline{B_s} \to D_s^+ K^-(t)}{B_s \to D_s^+ K^-(t) + \overline{B_s} \to D_s^+ K^-(t)}.$$  

This has the following dependence:

$$\mathcal{A}_{CP}(D_s^+ K^-)(t) = \frac{C_s \cos \Delta m_s t + S_s \sin \Delta m_s t}{\cosh(\Delta \Gamma_s t/2) - A_{\Delta \Gamma_s} \sinh(\Delta \Gamma_s t/2)},$$

where $\Delta m_s$ and $\Delta \Gamma_s$ are the mass and lifetime difference between the heavy and light $B_s$ eigenstates, which for the purposes of this discussion are assumed to be known. The three observables $C_s$, $S_s$ and $A_{\Delta \Gamma_s}$ can then be fitted from the data. (In doing this the full flavour untagged statistics may be used to fix $A_{\Delta \Gamma_s}$.) From performing an equivalent analysis for the $D_s^+ K^- \to D_s K^- \to D_s K^- \to D_s K^- \to D_s K^- \to D_s K^-$ final state three additional observables, $\overline{C_s}$, $\overline{S_s}$ and $\overline{A_{\Delta \Gamma_s}}$, can be obtained. The observables depend on the underlying physics parameters in the following manner:

$$C_s, \overline{C_s} = - (+) \left( \frac{1 - r_s^2}{1 + r_s^2} \right),$$

$$S_s, \overline{S_s} = \frac{2 r_s \sin(\phi_s + \gamma + (-) \delta_s)}{1 + r_s^2},$$

$$A_{\Delta \Gamma_s}, \overline{A_{\Delta \Gamma_s}} = - \frac{2 r_s \cos(\phi_s + \gamma + (-) \delta_s)}{1 + r_s^2}.$$  

Here $r_s$ is the ratio of amplitudes between the interfering tree diagrams, $\delta_s$ is a possible CP conserving strong phase difference between the diagrams and $\phi_s$ is the CP violating weak phase associated with the $B_s - \overline{B_s}$ oscillations, believed to be very small in the Standard Model. It is assumed that $\phi_s$ can be constrained from other measurements, such as those in $B_s \to J/\psi \phi$ decays.

Measurement of the six observables $C_s, \overline{C_s}, S_s, \overline{S_s}, A_{\Delta \Gamma_s}$ and $\overline{A_{\Delta \Gamma_s}}$ allows $r_s, \delta_s$ and $\gamma$ to be determined. The same information may be extracted by making a simultaneous event-by-event fit to the four decay distributions.

Exactly parallel relations hold in the $B_d$ system, with the analysis of $B_d \to D^\pm\pi^\mp$ or $B_d \to D^{*\pm}\pi^\mp$ decays. In this case however, the negligible width splitting between the mass eigenstates, mean that there are effectively only four observables: $C_d, \overline{C_d}, S_d$ and $\overline{S_d}$. These
observables depend on $r_d$, $\phi_d$ (measured from $B_d \to J/\psi K_s^0$ and equal to $-2\beta$ in the Standard Model), $\delta_d$ and $\gamma$. Note that the value of these observables will in general be different between $B_d \to D^\pm \pi^\mp$ and $B_d \to D^{*\pm} \pi^\mp$, firstly because of the possibility of different values of $r_d$ and $\delta_d$ for the two cases, and secondly because the $l = 1$ state of the $B_d \to D^{*\pm} \pi^\mp$ decay introduces some sign flips in expressions 2 and 3 (see [2] for more details).

3 LHCb Event Yields and Performance

LHCb has reported simulation studies of $B_s \to D_s^\pm K^\mp$ and $B_d \to D^{*\pm} \pi^\mp$ in [3]. The results are summarised here, together with estimates from a preliminary study of $B_s \to D_s^\pm K^\mp$.

The isolation of $B_s \to D_s^\pm K^\mp$ decays is experimentally very challenging, because of the low branching ratio and the order-of-magnitude more prolific $B_s \to D_s^\pm \pi^\mp$ decay mode. The LHCb trigger system gives good performance for fully hadronic modes and selects $B_s \to D_s^\pm K^\mp$ events with an efficiency of 30\%. The $\pi - K$ discrimination of the RICH system reduces the $B_s \to D_s^\pm \pi^\mp$ contamination to $\approx 10\%$. It is estimated that the experiment will accumulate 5.4k events per year of operation (with a year defined as 2 fb$^{-1}$ of integrated luminosity), with a combinatoric background to signal level of $< 0.5$. The excellent $\approx 40$ fs proper time precision provided by the silicon Vertex Locator will ensure that provided $\Delta m_s$ is not far in excess of expectation, the $B_s$ oscillations will be well resolved, and hence the CP asymmetries to be measured. It is estimated that the statistical precision on $\gamma$ from this channel alone will be $14^\circ$ for 2 fb$^{-1}$, assuming $\Delta m_s = 20$ ps$^{-1}$, $\Delta \Gamma_s/\Gamma_s = -0.10$ and taking plausible values of $\gamma$ and $\delta_s$.

The channel $B_d \to D^{*\pm} \pi^\mp$ has been investigated through the sub-decay $D^{*\pm} \to D^0 (\to K\pi) \pi^\mp$, in which it is estimated 206k events will be accumulated per year, with a background to signal level $< 0.3$. Earlier studies using inclusive $D^0$ decays [4] suggest that these statistics can be increased still further.

Studies are underway to investigate the feasibility of reconstructing $B_d \to D^\pm \pi^\mp$, with $D^\pm \to K^\pm \pi^\mp \pi^\pm$. The preliminary indications are that 210k reconstructed events will be collected each year, with a background to signal ratio of around 0.3.

4 Extraction of $\gamma$ from modes in isolation

In extracting $\gamma$ from the observables of a single decay mode, two problems are encountered, one general, and one specific to the $B_d$ decays.

1. The extraction of $\gamma$ from $S_{s(d)}$ and $\overline{S_{s(d)}}$ yields 8 possible solutions. The same is true of calculating $\gamma$ from $A_{\Delta \Gamma_s}$ and $\overline{A_{\Delta \Gamma_s}}$. Figure 1 illustrates this by plotting all possible solutions for $\gamma$ and $\delta_s$ in the case where the true values are assumed to be $\gamma = 60^\circ$ and $\delta = 60^\circ$. Therefore the study of $B_d \to D^\pm \pi^\mp$ or $B_d \to D^{*\pm} \pi^\mp$ in isolation results in an 8-fold ambiguity for $\gamma$. The extra observables available in the $B_s$ system mean that in principle there is only a 2-fold ambiguity, but in practice measurement errors may make it difficult to exclude local minima coming from the additional bogus solutions. This problem will be accentuated if the magnitude of $\Delta \Gamma_s/\Gamma_s$ is small, hence making $A_{\Delta \Gamma_s}$ and $\overline{A_{\Delta \Gamma_s}}$ difficult to measure.
Figure 1: Valid solutions for $\delta_s$ and $\gamma$ for the flavour tagged $S_s$, $\overline{S}_s$ observables (a) and the untagged $A_{\Delta\Gamma_s}$, $\overline{A}_{\Delta\Gamma_s}$ observables (b). In both cases there are eight solutions (although, in the case of (b), these are two-fold degenerate). The solid circle indicates the true solution $\gamma = 60^\circ$, $\delta_s = 60^\circ$.

2. As is clear from expressions 2 and 3, extracting $\gamma$ from $S_{s(d)}$ and $\overline{S}_{s(d)}$, or $A_{\Delta\Gamma_s}$ and $\overline{A}_{\Delta\Gamma_s}$, require that $r_{s(d)}$ be known. From comparing the CKM elements in the interfering diagrams it is expected that $r_s \sim 0.4$ and $r_d \sim 0.02$. Therefore $|C_s|$ will be significantly different from 1, and measurements of $C_s$ and $\overline{C}_s$ will allow $r_s$ to be extracted from the data – indeed this is what is done in the present LHCb simulation studies. This will not be possible, however, for $r_d$. Instead this parameter has to be set using external assumptions [5]. These assumptions introduce a troublesome systematic error to the analysis.

Both of these problems may be tackled by making a combined analysis of U-spin related $B_d$ and $B_s$ modes.

5 A Combined U-Spin Analysis of $B_s \rightarrow D_s^\pm K^\mp$
and $B_d \rightarrow D^\pm \pi^\mp$

The decays $B_s \rightarrow D_s^\pm K^\mp$ and $B_d \rightarrow D^\pm \pi^\mp$ are identical under U-spin symmetry, that is the exchange of $d$ and $s$ quarks. This symmetry allows the observables in both decays to be combined in a manner to yield certain relations, which then give $\gamma$ under the assumption that the a priori unknown hadronic contributions to the observables are identical in both channels. These unknowns are the strong phases $\delta_d$ and $\delta_s$, and $a_d$ and $a_s$, the hadronic contributions to $r_d$ and $r_s$, defined by:

$$r_{d,s} = a_{d,s} f_{d,s}^{CKM},$$

where the CKM factors, $f_{d,s}^{CKM}$, are easily calculable.

The following analysis follows the strategy proposed in [2]. The example plots and numbers assume the scenario $\gamma = 60^\circ$, $\delta_{d,s} = 60^\circ$, $a_{d,s} = 1$, $\phi_d = 47^\circ$ and $\phi_s = 0^\circ$. The experimental contours assume that in one year LHCb can measure $S_d$ and $\overline{S}_d$ with an uncertainty of 0.014, and $S_s$ and $\overline{S}_s$ with an uncertainty of 0.14 (results consistent with the performance figures quoted in
section 3). It is also useful to assume that in the early year of operation the analysis can benefit from studies of $B_d \to D^\pm \pi^\mp$ made at the B Factories. Taking existing measurements [6], and scaling to 2500 fb$^{-1}$ to represent a plausible B-factory integrated luminosity in 2008 gives an error on $S_d$ and $\overline{S}_d$ of 0.014.

Using expression 2, the sine observables for the $B_s$ and $B_d$ channels may be combined to give the following exact relations:

$$\frac{a_s \cos \delta_s}{a_d \cos \delta_d} = -\frac{1}{R} \frac{\sin(\phi_d + \gamma)}{\sin(\phi_s + \gamma)} \left( \frac{S_s + \overline{S}_s}{S_d + \overline{S}_d} \right),$$

$$\frac{a_s \sin \delta_s}{a_d \sin \delta_d} = -\frac{1}{R} \frac{\cos(\phi_d + \gamma)}{\cos(\phi_s + \gamma)} \left( \frac{S_s - \overline{S}_s}{S_d - \overline{S}_d} \right).$$

Here $R = \frac{1-r^2}{\lambda^2} \frac{1+r^2}{1+r_d^2}$, where $\lambda$ is the sine of the Cabibbo angle. Because $r_d << 1$ $R \approx \frac{1-r^2}{\lambda^2(1+r_d^2)}$ and hence these relations may be exploited without the need to measure $r_d$. In the limit of full U-spin symmetry, the left hand sides of equations 4 and 5 are equal to unity, and both relations give a determination of $\gamma$.

Figure 2: Contours formed from expression 4 (a) and expression 5 (b) showing the 1 sigma contours for one year of LHCb data, and one year of LHCb data together with the measurements which may be available from the B-factories by 2008. The solid circle indicates the true solution; the inverted solid triangles indicate fake solutions from an analysis of $B_s \to D_s^\pm K^\mp$ alone. Exact U-spin symmetry corresponds to a value of unity for the ordinate in both plots.

Figure 2 show the one sigma contours which will be obtained with one year of LHCb operation, together with the improvement possible if B-factory data are also included. The ambiguous solutions from the ‘conventional’ analysis are indicated. It can be seen that these bogus solutions are disfavoured by the combined analysis. There is, however, a mirror solution at $-180 + \gamma$ outside the region of the plots. This possibility can be rejected either by making assumptions
about the orientation of the unitarity triangle, or by noting that such a solution is accompanied by a very sizable strong phase difference, which is unlikely to be the case.

The precision achievable on $\gamma$ is in general different for the two expressions. In the chosen scenario, relation 5 gives the best result, returning an uncertainty of $\sigma_\gamma =^{+16}_{-7}$ degrees. In the one year analysis the contribution of the B-factory data is significant, if 2500 fb$^{-1}$ integrated luminosity is assumed. With five years of LHCb data (and 2500 fb$^{-1}$ from the B-factories) the precision improves to 5°.

The plots also allow any biases from U-spin breaking effects to be assessed. It can be seen that relation 5 has the steepest contour and hence exhibits the best robustness. For example, a 20% deviation in $a_s \cos \delta_s / a_d \cos \delta_d$ from unity leads to a 3° bias in $\gamma$.

In order to further assess the reliability the $\gamma$ extraction, the U-spin dependence may be weakened. This can be done by combining relations 4 and 5 into a single expression which involves either $\delta_d$ and $\delta_s$ or $a_s$ and $a_d$. For example, the latter exercise yields the equation

$$\frac{a_s}{a_d} = \pm \frac{1}{R} \sin 2(\phi_d + \gamma) \sqrt{\frac{(S^+_s)^2 \cos^2(\phi_s + \gamma) + (S^-_s)^2 \sin^2(\phi_s + \gamma)}{(S^+_d)^2 \cos^2(\phi_d + \gamma) + (S^-_d)^2 \sin^2(\phi_d + \gamma)}}. \tag{6}$$

It is now possible to determine $\gamma$ by demanding that $a_s = a_d$, but making no assumption about $\delta_d$ and $\delta_s$. Figure 3 shows the resulting one sigma contour for five years of data. The statistical precision is about 6 degrees. Again the dependence is sufficiently steep that deviations in $a_s/a_d$ from unity coming from U-spin breaking give relatively small biases in the result.

![Figure 3: Contour formed from expression 6 showing the 1 sigma contours for five years of LHCb data, together with the measurements which may be available from the B-factories by 2008. The solid circle indicates the true solution. U-spin symmetry for the magnitude of the $B_d$ and $B_s$ amplitudes corresponds to a value of unity for the ordinate.](image)

It should be emphasised that these analyses only make use of the flavour tagged observables in $B_s \to D_s^{\pm}K^\mp$ and $B_d \to D^{\pm}\pi^{\mp}$. If the magnitude of $\Delta \Gamma_s$ is sufficiently large, then measurements of the untagged observables $A_{\Delta \Gamma_s}$ and $\bar{A}_{\Delta \Gamma_s}$ will provide additional information which
will help further in the exclusion of ambiguous solutions, and add to the ultimate precision on \( \gamma \).

6 Conclusions

LHCb will accumulate large samples of \( B_s \to D_s^\pm K^\mp \), \( B_d \to D^{*\pm}\pi^\mp \) and \( B_d \to D^{\pm}\pi^\mp \) events. Independently each of these channels may be used to extract the CKM angle \( \gamma \), although in the case of the \( B_d \) channels this requires making assumptions about \( r_d \), the relative magnitude of the interfering tree diagrams. When using the flavour-tagged observables alone, this \( \gamma \) determination carries with it an 8-fold ambiguity, which compromises the usefulness of the measurement.

A combined analysis of \( B_s \to D_s^\pm K^\mp \) and \( B_d \to D^{\pm}\pi^\mp \) under the assumption of U-spin symmetry allows the true solution to be isolated with only a 2-fold ambiguity. Plausible assumptions about the size of the strong phase difference or the orientation of the unitarity triangle allow the remaining bogus solution to be excluded. This U-spin analysis has the further benefit of exploiting the \( B_d \) data without the need to know \( r_d \).

The intrinsic precision of the combined analysis is competitive with other approaches. For example, in the example scenario considered, a 5° uncertainty is achievable after five years of operation. The combined analysis does not make use of the untagged observables available in \( B_s \to D_s^\pm K^\mp \), which provide additional information which will improve the precision still further.

The systematic error associated with the assumption of U-spin symmetry can be transparently assessed through studying the contours associated with the measurements. In some cases these offer a very robust \( \gamma \) extraction. Furthermore, a variety of separate \( \gamma \) determinations may be performed, each with different U-spin symmetry assumptions. Comparison between the results will help in assigning the systematic error.

Finally, an analogous exercise can be performed from considering the U-spin related channels \( B_d \to D^{\pm}\pi^\mp \) and \( B_s \to D_s^{*\pm}K^\mp \). The reconstruction of the latter channel at LHCb is under investigation.

References


[4] J. Rademacker, Measuring the CKM Angle \( \gamma \) with \( B_d \to D^{*\pm}\pi^\mp \), CERN-LHCb-2001-153.

[5] For an overview of strategies, see Cecilia Voena’s talk at this conference.