Measuring $\gamma$ at LHCb with Dalitz Methods and Global Sensitivity

- Measuring $\gamma$ with $B^- \to D^0(K_S\pi\pi)K^-$
  - Yield and background expectations
  - Amplitude model fits
  - Model independent fits

- Other Dalitz Opportunities

- Global Sensitivity to $\gamma$ with Tree-Level Processes at LHCb

Guy Wilkinson (University of Oxford)
On behalf of the LHCb collaboration
Dalitz fits to extract $\gamma$
Measuring $\gamma$ in $B^- \rightarrow D^0(K_{S}\pi\pi)K^-$

Can measure $\gamma$ through the interference in $B^- \rightarrow (D^0/D^0)K^-$ decays…

\[ B^- \rightarrow D^0K^- \quad B^- \rightarrow \bar{D}^0K^- \]

…provided $D^0$ and $\bar{D}^0$ decay into a common final state.

B-factories have pioneered Dalitz analysis of $K_{S}\pi\pi$.

How well will this method work at LHCb?
Hadronic state such as $B^- \to D^0(K_S\pi\pi)K^-$ is well suited to capabilities of LHCb
At earliest (‘L0’) trigger level hadron high-$p_T$ trigger most important discriminator
Offline, signal separated from background through usual cuts, eg. :

- $p_T$ requirements; impact parameter significance w.r.t. primary-vertex;
- vertex chi2; RICH PID requirements; consistency of reconstructed B
  flight direction and direction of primary-vertex→B-vertex; mass cuts…

Only specific challenge w.r.t. other hadronic decays comes from $K_S$
Reconstruction. After all offline cuts ‘DD’ make up ~2/3 of sample.

Ongoing work focused at finding ‘DD’ tracks with necessary speed in HLT. Here assume
HLT selection is fully efficient.
Event Yields and Background

Mature analysis - numbers assumed for sensitivity studies

Expected yield in 2 fb$^{-1}$: 5000 events

Background studied with inclusive $b\bar{b}$ events, & with specific $B^−\rightarrow D^0\pi^−$ sample

- Combinatorics from $b\bar{b}$
  
  $B/S < 0.7$ at 90% CL

- Contamination from $B^−\rightarrow D^0\pi^−$
  
  $B/S \approx 0.25$

Indications from most recent simulation study

This number looks rather stable

Study with similar samples and with large $bb$ MC sample enriched in ‘dangerous’ events.

No indication to revise significantly this estimate; also have learnt dangerous ‘DK’ component (see later) is not dominant

$B^−\rightarrow D^0\pi^−$ background suppressed to < 10% through introducing max. momentum cut (good for RICH)
Acceptance and background classification

Dalitz acceptance rather flat and can be measured in data from $B^{-}\rightarrow D^{0}\pi^{-}$ events. Here model with polynomial for subsequent amplitude fit.

Make-up of background found in $b\bar{b}$ events largely unknown.

2 main possibilities:

- True $D$ with random/fake $K$ ('DK combinatorial')
- Fake $D$ ('Phase-space combinatorial')

Consider 3 scenarios with a 0.7 B/S coming from: 1) all 'DK combinatoric'; 2) half DK combinatoric, half phase-space; 3) all phase-space
Extracting $\gamma$

LHCb simulated sample for 2 fb$^{-1}$ (with no background), CPV included:

The $K_S^{0}\pi\pi$ Dalitz plots contain a CP-violating contribution from the $B^+$ and $B^-$ interference which is sensitive to $\gamma$. Consider two analysis methods:

- Unbinned fit based on amplitude model;
- Model independent binned fit using results from $\psi(3770)$ on D decays
LHCb has followed example of B-factories and investigated potential of amplitude fit to extract $\gamma$ from analysis of $K_S\pi\pi$ Dalitz plots from $B^+$ and $B^-$ Generate, and then fit, events assuming isobar model. Have considered both BaBar [PRL 95 (2005) 121802] and Belle [hep-ex/0411049] models. (Sensitivity results consistent, so here only report those from latter.)

Sensitivities for 2 fb$^{-1}$ (ie. 5000 events)

<table>
<thead>
<tr>
<th>Fit Scenario</th>
<th>$\sigma_\gamma$</th>
<th>$\sigma_{r_B}$</th>
<th>$\sigma_{\delta_B}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No background; flat acceptance</td>
<td>5.8$^\circ$</td>
<td>0.010</td>
<td>6.0$^\circ$</td>
</tr>
<tr>
<td>$D\pi$ (B/S=0.24) + phase-space (B/S=0.7); real acceptance</td>
<td>9.1$^\circ$</td>
<td>0.017</td>
<td>9.0$^\circ$</td>
</tr>
<tr>
<td>$D\pi$ + phase-space (B/S=0.35) + DK (B/S=0.35); real acceptance</td>
<td>9.8$^\circ$</td>
<td>0.018</td>
<td>9.3$^\circ$</td>
</tr>
<tr>
<td>$D\pi$ + DK (B/S=0.7); realistic acceptance</td>
<td>10.7$^\circ$</td>
<td>0.017</td>
<td>9.1$^\circ$</td>
</tr>
</tbody>
</table>

Input values:

$\gamma = 60^\circ; \delta_B=130^\circ; r_B=0.10$

Fits work well, with any bias small compared with statistical uncertainty

Results scale with sample sizes & $r_B$ in expected manner

So statistical error on $\gamma$ of 9-11$^\circ$. Total error will also include a model uncertainty – latest BaBar analysis [PRD 78 (2008) 034023] estimates this at 7$^\circ$. 

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Binned Model-Independent Fit


Number of events for flavour-tagged D sample

\[ N_{i}^{\pm} = h(K_{\mp i} + 2r_{B}^{2}K_{+i} + 2\sqrt{K_{i}K_{-i}}(x_{\pm c_{i}} \pm y_{\pm s_{i}})) \]

Choosing bins of similar strong phase difference maximises statistical precision

No model error! Instead: i) slight degradation in statistical precision; ii) residual error on \( \gamma \) from finite CLEO-c statistics:

- B & P [arXiv:0801.0840] estimate 5° (used in our global fit)
- Latest CLEO-c estimate is 1-2° [Asner, ICHEP 08]

Can be measured directly in quantum correlated decays at \( \psi(3770) \)! Expect final CLEO-c results soon…
Binned Fit: LHCb study  [LHCb-2007-141]

Study performed using BaBar isobar model  [PRL 95 (2005) 121802], both for generation and to define 8 equally separated bins in strong phase difference.

Include acceptance variation & background.

Fit parameters $x^\pm$ & $y^\pm$, then transform to physics parameters

Sensitivities for 2 fb$^{-1}$ (ie. 5000 events)

<table>
<thead>
<tr>
<th>Fit Scenario</th>
<th>$\sigma_\gamma$</th>
<th>$\sigma_{rB}$</th>
<th>$\sigma_{\delta B}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No background; flat acceptance</td>
<td>7.9$^\circ$</td>
<td>0.013</td>
<td>8.0$^\circ$</td>
</tr>
<tr>
<td>D$\pi$ (B/S=0.24) + phase-space (B/S=0.7); real acceptance</td>
<td>12.8$^\circ$</td>
<td>0.020</td>
<td>12.9$^\circ$</td>
</tr>
<tr>
<td>D$\pi$ + phase-space (B/S=0.35) + DK (B/S=0.35); real acceptance</td>
<td>12.8$^\circ$</td>
<td>0.020</td>
<td>12.6$^\circ$</td>
</tr>
<tr>
<td>D$\pi$ + DK (B/S=0.7); realistic acceptance</td>
<td>12.7$^\circ$</td>
<td>0.020</td>
<td>12.7$^\circ$</td>
</tr>
</tbody>
</table>

Compare with amplitude fit:
- $D\pi$+DK bckgd scenario 20% worse
- Results more robust against bckgd
Sensitivity to $\gamma$ with $\int L dt$

Calculate how total error evolves with time for amplitude fit & binned approach.

Error at end of baseline LHCb (10 fb$^{-1}$): 8.5° (amplitude model); 6.0° (binned).

These numbers neglect experimental systematics. For unbinned fit, resolution and acceptance effects have been considered and are expected to be small.
Sensitivity to $\gamma$ with $\int L dt$

In global fit studies (see later) a binned analysis is assumed with 5° CLEO error.

This is historical, and – if we believe ‘Asner ICHEP 08’ – conservative.

This assumption gives a 10 fb⁻¹ error on $\gamma$ of 7.6° (new CLEO-c value $\rightarrow$ 6.0°).
Other Opportunities in Dalitz Studies

$B^- \rightarrow D^0(K_S\pi\pi)K^-$ approach can be extended to other multi-body modes.

Some possibilities:

- $D^0 \rightarrow K_SKK$ (already exploited at BaBar [PRD 78 (2008) 034023]) or $D^0 \rightarrow K_SK\pi$
- $D^0 \rightarrow KK\pi\pi$: proposed in PLB 647 (2007) 400 and explored in LHCb in LHCb-2007-098. Same principle, but Dalitz space now requires 5-variables.
  Amplitude model now exists, [FOCUS, PRD B610 (2005) 225] but requires further development.
  LHCb error on $\gamma$: $18^o$ in 2 fb$^{-1}$
- Perhaps Dalitz fits of suppressed ADS modes for $K\pi\pi$ or $K\pi\pi^0$?
Combined Fit to $\gamma$
LHCb Global Sensitivity to \( \gamma \)

With B\( \to \)DK methods perform global fit of common parameters. In addition consider results from B\( ^0 \) and Bs time dependent analyses. (LHCb-2008-031)

Input measurements considered:

\[
\text{B}^+ \to \text{D}^0 \text{K}^-:
\]
- \( \text{D}^0 \to \text{K}\pi, \text{KK}, \pi\pi \) (LHCb-2008-011) \\
- \( \text{D}^0 \to \text{K}\pi\pi\pi \) (LHCb-2007-004) \\
- \( \text{D}^0 \to \text{K}_S\pi\pi \) (LHCb-2007-048)

\[
\text{B}^0 \to \text{D}^0 \text{K}^{*0}
\]
- \( \text{D}^0 \to \text{K}\pi, \text{KK}, \pi\pi \) (LHCb-2007-050)

Time dependent measurements:
- \( \text{B}^0 \to \text{D}\pi \) (LHCb-2007-044) \\
- \( \text{B}_s \to \text{D}_s \text{K} \) (LHCb-2007-041)

Summary of event yields in 2 fb\(^{-1}\)

<table>
<thead>
<tr>
<th>Channel</th>
<th>Signal</th>
<th>Background</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{B}^\pm \to \text{D}(\text{K}^\pm\pi^\mp)\text{K}^\pm )</td>
<td>56k</td>
<td>35k</td>
</tr>
<tr>
<td>( \text{B}^+ \to \text{D}(\text{K}^+\pi^+)\text{K}^+ )</td>
<td>650</td>
<td>780</td>
</tr>
<tr>
<td>( \text{B}^- \to \text{D}(\text{K}^-\pi^-)\text{K}^- )</td>
<td>400</td>
<td>780</td>
</tr>
<tr>
<td>( \text{B}^+ \to \text{D}(\text{K}^\mp\text{K}^\pm + \pi^+\pi^-)\text{K}^\mp )</td>
<td>3.3k</td>
<td>7.2k</td>
</tr>
<tr>
<td>( \text{B}^- \to \text{D}(\text{K}^+\text{K}^- - \pi^+\pi^-)\text{K}^- )</td>
<td>4.4k</td>
<td>7.2k</td>
</tr>
<tr>
<td>( \text{B}^\pm \to \text{D}(\text{K}^+\text{K}^- \pi^+\pi^-)\text{K}^\mp )</td>
<td>61k</td>
<td>40k</td>
</tr>
<tr>
<td>( \text{B}^+ \to \text{D}(\text{K}^\pm\pi^+\pi^-)\text{K}^\mp )</td>
<td>470</td>
<td>1.2k</td>
</tr>
<tr>
<td>( \text{B}^- \to \text{D}(\text{K}^+\text{K}^- \pi^+\pi^-)\text{K}^- )</td>
<td>350</td>
<td>1.2k</td>
</tr>
<tr>
<td>( \text{B}^0 \to \text{D}(\text{K}^\mp\text{K}^\pm)\text{K}^\pm )</td>
<td>34k</td>
<td>1/4k</td>
</tr>
<tr>
<td>( \text{B}^0 \to \text{D}(\text{K}^+\text{K}^-)\text{K}^\pm )</td>
<td>350</td>
<td>850</td>
</tr>
<tr>
<td>( \text{B}^0 \to \text{D}(\text{K}^+\text{K}^-)\text{K}^\pm )</td>
<td>230</td>
<td>850</td>
</tr>
<tr>
<td>( \text{B}^0 \to \text{D}(\text{K}^+\text{K}^- + \pi^+\pi^-)\text{K}^\pm )</td>
<td>190</td>
<td>600</td>
</tr>
<tr>
<td>( \text{B}^0 \to \text{D}(\text{K}^+\text{K}^- - \pi^+\pi^-)\text{K}^\pm )</td>
<td>550</td>
<td>500</td>
</tr>
<tr>
<td>( \text{B}^\pm \to \text{D}(\text{K}^0\text{K}^\pm\pi^\pm\pi^\mp)\text{K}^\mp )</td>
<td>5k</td>
<td>4.7k</td>
</tr>
<tr>
<td>( \text{B}_s, \bar{\text{B}}_s \to \text{D}_s^\pm \text{K}^\mp )</td>
<td>6.2k</td>
<td>4.3k</td>
</tr>
<tr>
<td>( \text{B}^0, \bar{\text{B}}^0 \to \text{D}^+\pi^\pm )</td>
<td>1.300k</td>
<td>290k</td>
</tr>
</tbody>
</table>

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**B→DK free parameters and constraints**

**Free parameters (and values used toy MC studies)**

Parameters common to \(B^-\to DK^-\):

- \(r_B\) - ratio of magnitude of diagrams (0.1)
- \(\delta_B\) - strong phase difference (130°)

\(B^0\to DK^{0*}\) analogues: \(r_{B^0}\) (0.40), \(\delta_{B^0}\) (scan)

D decay parameters for \(K\pi\), \(K\pi\pi\):  

- \(\delta_{K\pi}^D\) (-158°), \(\delta_{K^3\pi}^D\) (144°) - strong phase differences (\(r_{K\pi}^D\), \(r_{K^3\pi}^D\) well known)

- \(R_{K^3\pi}\) - coherence factor

\[
\Gamma(B^- \to (K^+\pi^-\pi^+\pi^-)_D K^-) \propto r_D^2 + (r_{K^3\pi}^D)^2 + 2r_B r_{K^3\pi}^D R_{K^3\pi} \cos(\delta_B + \delta_{K^3\pi}^D - \gamma)
\]

And of course \(\gamma\) (60°)

**Constraints (from CLEO-c)**

- PRL 100 (2008) 221801:  
  \[\delta_{D}^{K\pi} = (-158 ^{+22}_{-16})^{\circ}\]
  (ADS formalism requires -180° phase shift w.r.t. published result)

- Preliminary: arXiv:0805.1722

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Fitted $B \to DK$ parameters for many 2fb$^{-1}$ experiments

In general Gaussian – tails less pronounced than in fits to individual modes.

![Histograms showing fitted parameters](image)

Gaussian fit $\sigma_{\gamma} = 5.7^\circ$
Include Time Dependent CP Measurements

Add results from time dependent CP-measurements (see Carbone talk):

\( B_s \rightarrow D_s K \) very powerful \([LHCb-2007-041]\) :

As explained in Carbone talk, \( B^0 \rightarrow D \pi \) is very promising, but in conventional analysis requires complementary measurement, eg. \( B^0 \rightarrow D^* \pi \), or alternatively \( U \)-spin combination with \( B_s \rightarrow D_s K \) to yield competitive results \([LHCb-2008-035]\).

Here take \( B^0 \rightarrow D \pi \) uncertainty of 20° in 2 fb\(^{-1}\) (rather conservative).
Sensitivity to $\gamma$ including all measurement

Results shown as function of $\delta_{B^0}$, least well known parameter. Sensitivity of $B^0 \to D^0 K^{*0}$ improves by factor of two in going from $\delta_{B^0} = 45 \to 180^\circ$. Residual dependence remains in global fit, but diluted due to other measurements.

<table>
<thead>
<tr>
<th>$\delta_{B^0}$ ($^\circ$)</th>
<th>0</th>
<th>45</th>
<th>90</th>
<th>135</th>
<th>180</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\gamma}$ for 0.5 fb$^{-1}$ ($^\circ$)</td>
<td>8.1</td>
<td>10.1</td>
<td>9.3</td>
<td>9.5</td>
<td>7.8</td>
</tr>
<tr>
<td>$\sigma_{\gamma}$ for 2 fb$^{-1}$ ($^\circ$)</td>
<td>4.1</td>
<td>5.1</td>
<td>4.8</td>
<td>5.1</td>
<td>3.9</td>
</tr>
<tr>
<td>$\sigma_{\gamma}$ for 10 fb$^{-1}$ ($^\circ$)</td>
<td>2.0</td>
<td>2.7</td>
<td>2.4</td>
<td>2.6</td>
<td>1.9</td>
</tr>
</tbody>
</table>

Weight (in %) of each contributing analysis with 2 fb$^{-1}$ for two values of $\delta_{B^0}$:

<table>
<thead>
<tr>
<th>Analysis</th>
<th>$\delta_{B^0} = 0^\circ$</th>
<th>$\delta_{B^0} = 45^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^- \to D^0(h\bar{h})K^-$, $B^- \to D^0(K^{+\pi^-\pi^-\pi^-})K^-$</td>
<td>25</td>
<td>38</td>
</tr>
<tr>
<td>$B \to D^0(K_S^{0}\pi^+\pi^-)K$</td>
<td>12</td>
<td>25</td>
</tr>
<tr>
<td>$B^0 \to D^0(h\bar{h})K^{*0}$</td>
<td>44</td>
<td>8</td>
</tr>
<tr>
<td>$B_s \to D_S^{+}K^{\perp}$</td>
<td>16</td>
<td>24</td>
</tr>
<tr>
<td>$B^0 \to D_S^{+}K^+$</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>
Conclusions

• Measurement of $\gamma$ with $B^- \to D^0(K_S\pi\pi)K^-$ as pioneered by B-factories appears a very promising strategy at LHCb. Model independent approach gives uncertainty of $6^o$ over lifetime of baseline experiment.

• Dalitz strategy can be extended to other modes, eg: $K_S KK$, $KK\pi\pi$

• Combined fit of all tree-level $\gamma$ measurements helps obtain Gaussian fit results for parameters, and yields best-possible overall precision.

  $\rightarrow \gamma$ uncertainty around $2^o$ with $10$ fb$^{-1}$

• Although real data may inevitably hold nasty surprises, and final stage of HLT is still under development, there are also promising other modes not yet included in average/study:

  $\rightarrow D \to K\pi\pi^0, K_S KK, KK\pi\pi \ldots$, $B \to D^{(*)}K^{(*)} +$ other time dependent measurements, eg. $B^0 \to D^*\pi$, $B_s \to D_s^{(*)}K_1 +$ improved Information from other experiments (eg. $c_i$, $s_i$ precision from CLEO-c)

The work will begin very soon – first experience in reconstructing real hadronic final states in the next couple of months!
ADS/GLW $B^\pm$ measurements alone: the role of the external constraints

2 fb$^{-1}$ of data: $D^0(K\pi,KK,\pi\pi)K + D^0(K\pi\pi\pi)K +$ CLEO-c constraints; scan in $\delta_D^{K\pi}$

External constraints important: equivalent to doubling of B dataset at $\delta_D^{K\pi} = -158^\circ$
(And external input essential for $D^0 \rightarrow K_S\pi\pi$ to avoid model dependent systematic)